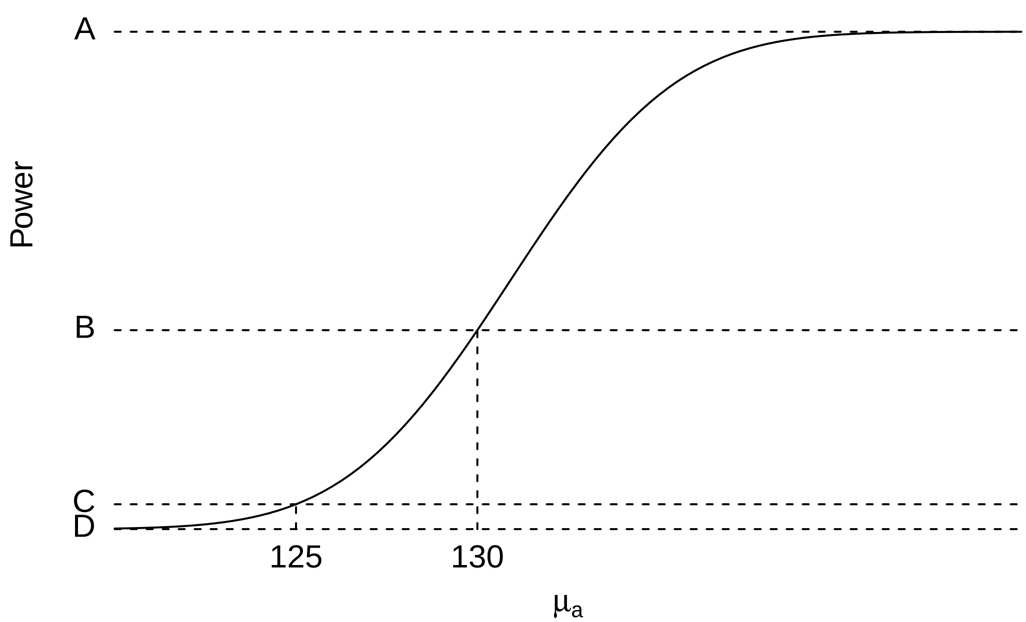
Economics 250 — Midterm 2 **14 November 2013**

Instructions:

- The exam is 80 minutes in length.
- You may use a hand calculator.
- Hand in your answers. Do not hand in the question and formula sheets.
- Show your work: incorrect answers without any work shown cannot be given partial marks.
- Answer **three** of the **four** questions in the answer booklet provided. If you attempt all four questions, indicate clearly which questions you wish to be graded. If there is no clear indication, only the first three answered questions will be graded.
- Each of the four questions is worth the same amount. The allocation of points within each question is as shown.
- This midterm has 4 pages, including this cover sheet. An additional 6 pages (3 formula sheet pages and 3 statistical tables) are also provided.

1. **[25]** You intend to collect a set of data to estimate μ, the population mean of X, a measure of happiness. You believe, before looking at the data, that the population mean is 125. $\sigma_X = 17.97$ is known.

Before performing the test, you decide on a significance level $\alpha = 0.05$ and sample size $n = 25$. You wish to examine the power of the z test statistic at an economically significant alternative that the mean has changed to 130, and calculate a test power of 0.4. Concerned that your test lacks power, you decide to calculate and graph the power (the probability of rejecting H_0 when H_a is true) of the test statistic for different specific values of μ_a:



a) **[4]** Write down the null (H_0) and alternative (H_a) hypotheses that agree with the graph above. Is this a one-tailed or two-tailed test?

b) **[4]** Write down the values of A, B, C, and D.

c) **[5]** What is the probability of making a Type I error (rejecting H_0 when H_0 is true)? What is the probability of making a Type II error (failing to reject H_0 when H_a is true) at the specific alternative $\mu_a = 130$?

d) **[6]** Would the test power increase, decrease, or stay the same if you calculated the power at $\mu_a = 135$, leaving the other parameters unchanged? Would the *graph* of test power change? If the graph changes: will it be above, equal to, or below B at $\mu_a = 130$; and will it be above, equal to, or below C at $\mu_a = 125$?

e) **[6]** Would the test power at $\mu_a = 130$ increase, decrease, or stay the same if you reduce the size of the sample from $n = 25$ to $n = 15$, leaving the other parameters unchanged? Would the *graph* of test power change? If the graph changes: will it be above, equal to, or below B at $\mu_a = 130$; and will it be above, equal to, or below C at $\mu_a = 125$?

2. **[25]** Suppose that passing Economics 250 increases a person's future earning potential with probability 0.75.

a) **[4]** If you select ten random students who passed Economics 250, what is the probability that exactly seven of them will have higher future incomes?

b) **[6]** What is the probability that, of the ten students selected in part *a*), at least eight of them will have higher future incomes?

c) **[5]** What is the probability that, of the ten students selected in part *a*), six or fewer will have higher future income? *Hint: using your answers from parts* a*) and* b*) may help.*

d) **[5]** If you select eighty (80) students who passed Economics 250, what is the probability that at least 55 of them will have higher future incomes? Your answer should use an appropriate approximation.

e) **[5]** Of the eighty students selected in part *d*), what is the probability that at least 60 will have higher future incomes? Your answer should use an appropriate approximation.

3. **[25]** A researcher is analyzing the returns of investing in a stock of a hot new company and collects daily data on the stock's performance, which he believes to be normally distributed. From a sample of the daily price over 15 days, he calculates the mean daily percentage change in the stock return to be 0.5.

a) **[6]** The researcher assumes that the variance of the measured variable equals $\sigma^2 = 10$ (and so standard deviation is $\sigma = 3.162$). Using this assumption, find a 97% confidence interval for the mean daily percentage stock return.

b) **[6]** A reporter doing a news story on the company for a local paper interviews the researcher. Upon learning the confidence interval, the reporter responds that the decimal places in the researcher's confidence will confuse his readers (and his editor). The reporter insists on using the narrower confidence interval $[-1, 2]$ instead of what you calculated above.

What confidence level must the reporter use to make the $[-1, 2]$ confidence interval statistically valid? *Hint: start by finding the critical value that would give the desired confidence interval endpoints.*

c) **[7]** Another researcher believes that the first researcher just made up the assumption of $\sigma^2 = 10$. She uses the same 15 observations to calculate $s^2 = 5.92$ (and so $s = 2.433$). Find a 95% confidence interval for the mean daily stock return without relying on the first researcher's assumption.

d) **[6]** If the reporter still wished to use the $[-1, 2]$ confidence interval with the second researcher's analysis, would the confidence level be greater or less than 95%?

4. [**25**] A researcher collects data on a country's mean monthly household income level in the month following a major increase in the country's minimum wage rate. She obtains a random sample of 12 of the country's households, collecting household income from each.

Thinking back to her Economics 250 course, the researcher remembers that income is usually strongly right-skewed, but that the logarithm of income is reasonably close to normal. She calculates $x_i = \ln(income_i)$ for each of the 12 households then uses these values to calculate the sample mean, $\overline{x} = 8.700$, and sample variance, $s^2 = 0.0963$ (and thus sample standard deviation $s = 0.310$).

She wishes to use the data to test the null hypothesis that mean household income has not changed from the pre-increase mean of $\mu = \ln(5166) = 8.55$ against the alternative that the mean household income has increased.

a) [**4**] State the null and alternative hypotheses the researcher wishes to test. State the hypotheses in terms of the log-income values (as given above) rather than direct income values.

b) [**5**] Find the test statistic associated with the test, and give as much information about the p-value as is possible given the available statistical tables.

c) [**4**] Can the researcher conclude that income has increased at the $\alpha = 0.05$ significance level? Can the researcher conclude that income has increased at the 95% confidence level?

d) [**4**] Suppose that the researcher had actually collected the data from $n = 24$ instead of $n = 12$ households and obtained the same $\overline{x}$, s^2, and s values given above. Would the test statistic and p-value change from what you calculated in part *b*)? If so, would they become larger or smaller?

e) [**3**] In order to calculate a (two-tailed) confidence interval for the mean household income, you would need to find a critical value, t^*. Write down the critical value you would use to calculate a 98% confidence interval for a data set of $n = 18$ households.

f) [**5**] If someone gave you a 99% confidence interval for μ of $[8.583, 8.817]$, would you reject $H_0 : \mu = 8.55$ against the two-sided alternative $H_a : \mu \neq 8.55$ at the $\alpha = 0.01$ significance level? At the $\alpha = 0.05$ significance level?