Economics 250 — Midterm 1 **10 February 2015**

Instructions: You may use a hand calculator.

Do not hand in the question and formula sheets.

Answer all **four** questions in the answer booklet provided.

The grade weight of each question is shown. There are 80 points in total and 80 minutes to complete the exam. Allocate your time accordingly.

Show your work: incorrect answers without any work shown cannot be given partial marks. If using your calculator, it is acceptable to only write down the first few terms of a calculation.

Formulas and tables are provided at the end of the question pages; you may wish to detach these from the question pages for easier reference.

1. **[25]** You obtain the following sample of annual Canadian household food expenditures and household income, both in thousands of dollars:

Food spending	7.7	7.0	7.1	6.9	8.0	7.8	7.3	7.3	9.3	8.1
Income	71	69	68	66	70	75	71	80	94	72

a) **[7]** Find the mean, median, and mode of food spending.

b) **[3]** Find the first and third quartiles of food spending.

c) **[5]** Find the standard deviation of food spending.

d) **[5]** Using *only the first 5 observations* given in the table above, calculate the correlation between food spending and income, and interpret the value. You may use the following values, which were calculated using just the first 5 observations:

Food spending mean	7.34
Food spending variance	0.233
Income mean	68.8
Income variance	3.7

e) **[5]** Now suppose that the data is actually *grouped data* of means for the 10 Canadian provinces. Also suppose that you are actually interested only in the Western provinces, for which populations (in millions) are given in the table below. Calculate the mean food spending by households in the Western provinces.

	Manitoba	Saskatchewan	Alberta	B.C.
Food spending:	7.3	7.3	9.3	8.1
Population:	1.3	1.1	4.0	4.6

Answers:

Sorting the data first helps for the median/quartiles, and for picking out the mode:

6.9 7.0 7.1 7.3 7.3 7.7 7.8 8.0 8.1 9.3

a)

$$\begin{aligned}\overline{x} &= \frac{1}{n}\sum x_i = \frac{76.5}{10} = 7.65 \\ x_{\text{median}} &= \frac{7.3+7.7}{2} = 7.5 \\ x_{\text{mode}} &= 7.3\end{aligned}$$

b)

$$\begin{aligned}Q_1 &= 7.1 \\ Q_3 &= 8.0\end{aligned}$$

c)

$$
\begin{aligned}
s_x^2 &= \frac{1}{n-1}\sum(x_i - \overline{x})^2 \\
&= \frac{4.605}{9} \\
&= 0.5127 \\
s_x &= \sqrt{s_x^2} \\
&= 0.7153
\end{aligned}
$$

d) First we need to calculate the covariance, using either version of the formula:

$$
\begin{aligned}
s_{xy} &= \frac{1}{n-1}\sum(x_i - \overline{x})(y_i - \overline{y}) \\
s_{xy} &= \frac{1}{n-1}\left(\left(\sum(x_i y_i)\right) - n\overline{xy}\right)
\end{aligned}
$$

The second is slightly easier to calculate, so using it:

$$
\begin{aligned}
s_{xy} &= \frac{1}{4}(2527.9 - 5(7.34)(68.8)) \\
&= 0.735
\end{aligned}
$$

Now we can plug this and the given variance values into the correlation formula to get:

$$
\begin{aligned}
r_{xy} &= \frac{s_{xy}}{s_x s_y} \\
&= \frac{0.735}{\sqrt{0.233}\sqrt{3.7}} \\
&= 0.7916
\end{aligned}
$$

Because this value is relatively close to 1, we can conclude that there is a strong, positive, linear relationship between food expenditure and income.

e) The question is basically just asking for a weighted sum calculation, using populations as the weights. We did a very similar example to this in class.

Using c_i to denote the population counts and m_i to denote the means:

$$
\begin{aligned}
\overline{x} &= \frac{\sum c_i m_i}{\sum c_i} \\
&= \frac{1.3 \times 7.3 + 1.1 \times 7.3 + 4.0 \times 9.3 + 4.6 \times 8.1}{1.3 + 1.1 + 4.0 + 4.6} \\
&= \frac{91.98}{11.0} \\
&= 8.3618
\end{aligned}
$$

Note that this is *not* the same as simple mean (which gives food spending values equal weight) of 8.0.

2. [**18**] Suppose that the exchange rate of the Canadian to US dollars, denoted X, is a random variable with mean 1.24 and standard deviation 0.02 (that is, 1 US dollar is worth, on average, \$1.24 Canadian). The exchange rate of Canadian dollars to euros, denoted Y, has mean 1.40 and standard deviation 0.03 (1 euro is worth \$1.40 Canadian on average). The covariance between these two random variables is 0.00045.

a) [**3**] Suppose that you find a US 20-dollar bill and add it to your wallet, which also contains 50 dollars in Canadian currency. Let W be the random variable of the Canadian dollar value of cash in your wallet. Write down an expression for W in terms of X (and any needed constants).

b) [**6**] Find the mean and standard deviation of W.

c) [**3**] Suppose that a friend is in desperate need for Canadian dollars, and gives you 50 euros in exchange for your 50 Canadian dollars. Write down an expression for V, the random variable corresponding to the Canadian dollar value of your wallet (which now contains 20 US dollars and 50 euros).

d) [**6**] Calculate the mean and standard deviation of V.

Answers:

a) $W = 50 + 20X$

b)

$$\begin{aligned}
\mu_W &= 50 + 20\mu_X = 50 + 20(1.24) = 74.80 \\
\sigma_W^2 &= 0 + 20^2\sigma_X^2 = 400(0.02^2) = 0.16 \\
\sigma_W &= \sqrt{\sigma_W^2} = \sqrt{0.16} = 0.4
\end{aligned}$$

c) $V = 20X + 50Y$

d) Use the "Functions of Random Variables" mean and variance formulas:

$$\begin{aligned}
\mu_V &= 20\mu_X + 50\mu_Y = 20(1.24) + 50(1.4) = 94.80 \\
\sigma_V^2 &= 20^2\sigma_X^2 + 50^2\sigma_Y^2 + 2(20)(50)\sigma_{XY} \\
&= 400(0.02^2) + 2500(0.03^2) + 2000(0.00045) \\
&= 3.31 \\
\sigma_V &= \sqrt{\sigma_V^2} = \sqrt{3.31} = 1.819
\end{aligned}$$

3. [**17**] Suppose that a friend of yours works at a popular bar, where on any given night her earnings are uniformly distributed between \$100 and \$300. Note that this distribution has mean of \$200 and standard deviation of \$57.7.

Find the probability that, on a random night, your friend earns:

a) [**2**] More than \$200.

b) **[3]** Between \$170 and \$230.

c) **[2]** Exactly \$200 (assume earnings are not rounded off to dollars/cents).

Your friend is scheduled to work 16 shifts this month. The earnings of each shift is independent, and is distributed as given above.

d) **[2]** Write down the (approximate) distribution of your friend's average earnings over the 16 shifts.

e) **[4]** Find the probability that your friend will earn an average of less than \$170 per night.

f) **[4]** Find the probability that your friend will earn an average of more than \$220 per night.

Answers:

Let $X \sim U[100, 300]$ be the distribution of earnings.

a) $P(X > 200)$: This is just asking for the probability of a value above the mean, which for a uniform distribution (or a normal, or any other distribution with its mean equal to its median) is 0.5.

You can also work this out by calculating the height of the distribution $1/(300 - 100) = 0.005$ and multiplying this by the width of the region being requested, which is $300 - 200$, so the probability is $(0.005)(100) = 0.5$.

b) $P(170 < X < 230)$: this is the region with height 0.005 and width $230 - 170 = 60$, and so has probability $(0.005)(60) = 0.3$.

c) $P(X = 200) = 0$, because this is a continuous distribution.

Now let $\overline{x} = \frac{1}{16} \sum_{i=1}^{16} X_i$, i.e. the average earnings over the 16 shifts.

d) This is just asking you to give the central limit theorem formula. Here n isn't particularly large, but as we saw, the mean of a sample from a uniform distribution gets pretty close to normal pretty fast. Thus:

$$\overline{x} \sim N\left(200, \frac{57.7}{\sqrt{16}}\right) \sim N(200, 14.425)$$

e) Convert to a Z score and look it up in the table:

$$P(X \leq 170) = P\left(Z \leq \frac{170 - 200}{14.425}\right) = P(Z \leq -2.08) = 0.0188$$

f) We can convert to a Z score first:

$$P(X > 220) = P\left(Z > \frac{220 - 200}{14.425}\right) = P(Z > 1.39)$$

Now you can either flip the inequality direction by subtracting from 1 (i.e. applying the complement rule) and looking up the result in the Z table:

$$P(Z > 1.39) = 1 - P(Z \leq 1.39) = 1 - .9177 = .0823$$

Or you can use the symmetry of the normal distribution to note that:

$$P(Z > 1.39) = P(Z < -1.39) = .0823$$

4. [**20**] Suppose that there is an ice cream shop on a beach. On any given day, the seller has either high, medium, or low profits. Let H denote high profits, M denote medium profits, and L denote low profits. On a given day it is either sunny (event S) or not ($S^{\complement}$).

The ice cream seller makes the following observations:

- It is sunny with probability 0.6.
- On sunny days: the profits are high with probability 0.7 and medium with probability 0.2.
- On non-sunny days: the profits are medium with probability 0.35 and low with probability 0.5.

a) [**5**] Write down the 5 probability values given above as probability statements expressed in terms of the H, M, L, and S events. (For example, $P(X|Y) = 0.123$ is a probability statement in terms of X and Y.)

b) [**3**] Find $P(S^{\complement})$

c) [**4**] Express and calculate the probability that a given day is sunny *and* has high profits.

The probability that the seller has high profits is 0.48 (you do not need to show this, but it may be helpful for the remaining parts of the question).

d) [**4**] Express and calculate the probability that a given day is sunny *or* has high profits.

e) [**4**] Determine whether or not being sunny and having high profits are independent events. Be sure to clearly indicate a condition that is satisfied or violated to support your answer.

Answers:

a)

$$\begin{aligned} P(S) &= 0.6 \\ P(H|S) &= 0.7 \\ P(M|S) &= 0.2 \\ P(M|S^{\complement}) &= 0.35 \\ P(L|S^{\complement}) &= 0.5 \end{aligned}$$

b) This is just the complement rule: $P(S^{\complement}) = 1 - P(S) = 1 - 0.6 = 0.4$.

c) Apply the multiplication rule: $P(S \cap H) = P(H|S)P(S) = (0.7)(0.6) = 0.42$

d) Applying the addition rule: $P(S \cup H) = P(S) + P(H) - P(S \cap H) = 0.6 + 0.48 - 0.42 = 0.66$.

e) There are three ways to show this: either of $P(H|S) \neq P(H)$, $P(H \cap S) = P(H)P(S)$. The first just uses given values; the second uses given values plus the answer to part *c*). The third way is to show $P(S|H) \neq P(S)$, but that requires applying Bayes's Rule to get $P(S|H)$, which wasn't in the material covered by this midterm.