Economics 250 — Midterm 2   **19 March 2015**

Instructions: You may use a hand calculator.

Do not hand in the question and formula sheets.

Answer all **five** questions in the answer booklet provided.

The grades available for each question is shown. There 80 points in total and 80 minutes to complete the exam. Allocate your time accordingly.

Show your work: incorrect answers without any work shown cannot be given partial marks. If using your calculator, it is acceptable to only write down the first few terms of a calculation.

Formulas and tables are provided at the end of the question pages; you may wish to detach these from the question pages for easier reference.

1. **[18]** A test for the presence of heart disease gives a numerical score denoted x.

   A person receives a positive diagnosis if $x > 90$.

   a) Among people *with* heart disease, x follows a normal distribution with mean 105 and standard deviation 8.1. What is the probability that a randomly selected person who has heart disease receives a positive diagnosis?

   b) Among people *without* heart disease, x follows a normal distribution with mean 70 and standard deviation 15.0. What is the probability that a randomly selected person who does not have heart disease receives a positive diagnosis?

   c) Suppose that it is known that 15% of the population has heart disease. Find the probability that a person who receives a positive diagnosis from this test actually has heart disease.

2. **[20]** A professor is interested in the study habits of his students and conducts a random sample of 20 students to determine the mean number of hours spent studying per week. This sample has a mean of 18.4. Suppose that $\sigma = 4.7$ is known and that the population is normally distributed.

   All parts of this question should be done at the $\alpha = 0.05$ level.

   a) Test the hypothesis that the mean number of hours spent studying per week is 20 against the alternative that the mean is less than 20. What is the p-value of this test?

   b) What is the power of this test when the population mean equals 18? What is the power when the population mean equals 20? What is the power when the population mean equals 21.9?

   c) What is the probability of making a Type I error with this test? What is the probability of making a Type II error when the population mean equals 18?

   Hint: Recall that a Type I error is rejecting the null hypothesis when it is true, and a Type II error is failing to reject the null hypothesis when the alternative hypothesis is true.

3. **[12]** Suppose that 70% of cars contain only one occupant (the driver); the rest contain at least two occupants.

   a) If you observe 10 cars, what is the probability that at least 7 of the cars have only one occupant? What is the probability that at least 8 of the cars have only one occupant?

   b) If you observe 100 cars, what is the probability that at least 70 have only one occupant? What is the probability that at least 80 of the cars have only one occupant?

4. **[18]** A researcher interested in the incomes of Canadian households has conducted a simple random sample of 45 Canadian households. This sample has a mean of $84,500 and standard deviation of $49,000.

   a) Assume that the population distribution is normal. Find a 95% confidence interval for the average Canadian household income.

   b) Still assuming that the population distribution is normal, perform a two-sided test at the 5% significance level that the population mean equals $75,000.

The researcher has strong doubts that the population is normal, and is particularly concerned that the data contains two outliers of $321,000 and $512,000. In fact, only 12 of the 45 sample values are above $75,000.

   c) Use a sign test to test the two-sided hypothesis that the median equals $75,000 at the 1% significance level. What is the *p*-value of this test?

5. **[12]** A researcher wishes to test whether house prices in a particular city changed from 2013 to 2014. She uses a random sample of the selling prices of 32 houses in 2013; the sample has a mean of $430,000 and standard deviation of $94,000. A similar sample of 40 houses sold in 2014 has a mean of $485,000 with sample standard deviation of $115,000. The researcher checks the data and verifies that neither sample contains any significant outliers.

   a) Write down the null and alternative hypotheses and perform the test at the 95% confidence level. What is the *p*-value of your test?

   b) Calculate a 99% confidence interval for the change in house prices from 2013 to 2014.