

Instructions: You may use a hand calculator.

Do not hand in the question and formula sheets. You can keep (or discard) these.

Answer all **three** questions in the answer booklet provided.

There are 80 points in total and 80 minutes to complete the exam.

Show your work: incorrect answers without any work shown cannot be given partial marks. If using your calculator, it is acceptable to only write down the first few terms of a calculation.

Formulas and tables are provided at the end of the question pages; you may detach these from the question pages for easier reference.

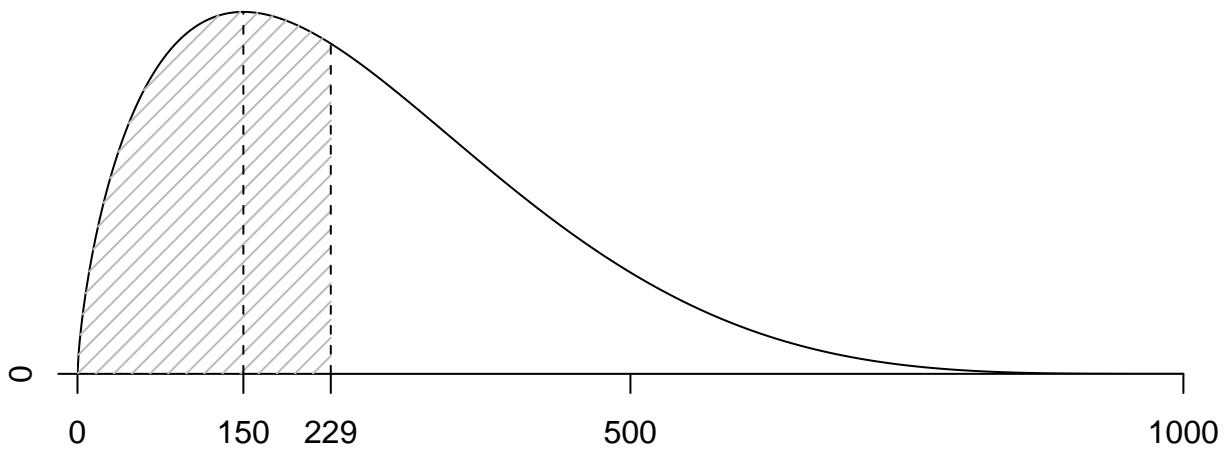
Good luck!

1. [20] The following table shows box office revenues and production costs for a sample of five recent movies (in millions of dollars).

Box office revenue	63	262	128	300	645
Production costs	90	136	9	95	282

- a) Calculate the mean and standard deviation of box office revenue.
- b) The mean and variance of production costs are 122.4 and 10079.3, respectively. Calculate the correlation between box office revenue and production costs.

Suppose box office revenue is known to always be between 0 and 1000 and follows the distribution with density shown below, where the shaded region is exactly half of the area under the curve.



- c) What is the area under the curve between 229 and 1000?
- d) Is this distribution left-skewed, symmetric, or right-skewed?
- e) What are the mean, mode, and median of this distribution? If one or more of these cannot be determined directly from the graph, indicate where the value would be relative to the given values (for example “between 500 and 1000”).

2. [30] An investor is considering making an investment of \$1000 into a stock. Suppose that X represents the dollar value of the investment after one year, and is known to be uniformly distributed between 900 and 1200, with standard deviation of 86.60.

- a) What is the mean value of the investment after a year?
- b) What is the probability that the investor loses value (that is, that the value after a year is less than the initial investment)?
- c) What is the probability of seeing a change in value of more than \$70 (either an increase *or* a decrease)?

As an alternative, the investor is considering making smaller investments of \$100 into each of 10 different stocks. The value of each of these investments after a year is known to be uniformly distributed with mean 105.

- d) If each of these investments has a value between 100 and 110 with probability 0.25, what are the minimum and maximum values of each investment?

Assuming the 10 stock values are independent of one another, the total value of the portfolio, W , will be very close to normally distributed with a mean of 1050 and standard deviation of 36.51. Use this approximation to answer the following questions.

- e) Which of the investments (the single investment X , or the diversified investment W) is more likely to be near its mean? How do you know?
- f) What is the probability that the investor loses value after a year with the diversified investment?
- g) What is the probability that the portfolio has a value between 1000 and 1100 after a year?

3. [30] A deck of standard playing cards has 52 cards. Each card has one of four suits (\spadesuit , \heartsuit , \diamondsuit , or \clubsuit) and one of thirteen ranks ($A, 2, 3, \dots, 9, 10, J, Q$, or K). Thus there are 13 cards of each suit, and 4 cards of each rank.

a) Suppose you draw one card from the deck at random. Let A denote the event that the card is a \spadesuit , and let B denote the event that the card is a 7. Find the following:

(i) $P(A)$ (ii) $P(B)$ (iii) $P(A^c)$ (iv) $P(A \cap B)$ (v) $P(B \cup A)$

b) Suppose you draw four cards at random from a deck, and look at them. The cards are: $5\heartsuit, Q\heartsuit, Q\spadesuit, 2\heartsuit$. Let C denote the event of drawing these 4 cards.

Let D denote the event that the next card you draw (i.e. the fifth card) is a Q . Write down a probability expression in terms of C and D that expresses the statement “the probability that, after drawing the four cards shown above, the next card is a Q ”, and find this probability.

c) Let E be the event that a card drawn at random is black (\spadesuit or \clubsuit), let F be the event that the card is a face card (J, Q , or K), and let G denote the event that the card is the $Q\spadesuit$.

(i) Find $P(G|E \cap F)$

(ii) Find $P(E \cap F|G)$

d) For each of the following pairs of events, indicate whether the two events are disjoint, and whether the two events are independent.

(i) A : the first card drawn is a \spadesuit .

B : the first card drawn is a black card (\spadesuit or \clubsuit).

(ii) A : the first card drawn has rank 3, 4, or 5.

B : the first card drawn is $6\diamondsuit$.

(iii) A : the first card drawn is a \spadesuit .

B : the first card drawn is a 7.

(iv) A : the *first* card drawn is a \diamondsuit .

B : the *second* card drawn is a \diamondsuit .

Now suppose that the deck is missing several cards. You know, however, that the probability of drawing a J is $1/8$, and that the probability of drawing a \heartsuit is $1/2$. You also know that if the card is a J there is a probability of $1/3$ that the card is a \heartsuit .

e) If a card drawn at random is a \heartsuit , what is the probability that the card is a J ?

f) Show that, with this deck, drawing a \heartsuit and drawing a J are *not* independent.