

Instructions: You may use a hand calculator.

Do not hand in the question and formula sheets.

Answer all **four** questions in the answer booklet provided.

The points available for each question is shown. There 40 points in total and 80 minutes to complete the exam. Allocate your time accordingly.

Show your work: incorrect answers without any work shown cannot be given partial marks.

Formulas and tables are provided at the end of the question pages; you may wish to detach these from the question pages for easier reference.

1. [9] Suppose that the value of 1 US dollar in Canadian dollars is normally distributed with mean 1.34 and standard deviation 0.02.

The value of 1 Euro in Canadian dollars is also normally distributed with mean 1.44 and standard deviation 0.03.

The correlation between these two exchange rates equals -0.3.

- a) Suppose that you have 30 US dollars, but you owe a friend 27 Euros. What is the mean value of your net assets (that is: the Canadian dollar value of what you have minus the Canadian dollar value of what you owe)?
- b) What is the distribution of the Canadian dollar value of your net assets?
- c) Find the probability that the Canadian dollar value of your net assets is positive.

2. [13] Suppose that a researcher interesting in studying household income in a developing country samples 20 of the country's households, calculating their income (converted to US dollars). The mean of her sample equals 3000.

Household income in the country is known to have a standard deviation of 1500.

The researcher is interesting in looking for statistically significant evidence in support of an increase in the population from its value ten years ago of 2500.

- a) Write down the hypotheses that the researcher wishes to test. Is this a one-tailed or two-tailed test?
- b) Find the p -value for this statistical test. Based on this value, can you reject, at the $\alpha = 0.1$ level, that there has been no increase in mean household income?
- c) What is the power of this test (with $\alpha = 0.1$) if the population mean income has increased by 300 (that is, if the population mean actually equals 2800)?
- d) Without changing α , how could the researcher have decreased the probability of making a Type II error (that is, failing to reject when rejecting is correct) when the true population mean equals 2800?

3. [8] Suppose that in a very boring class (not ECON 250, of course) held in the late afternoon, any student falls asleep with probability 0.3.
- a) If the class has 9 students, what is the probability that 2 or fewer students fall asleep?
 - b) If the class has 9 students, what is the probability that 3 or more students fall asleep?
 - c) If the class has 90 students, what is the probability that at least a third of the class will fall asleep?

4. [10] A potato farmer is trying out a new variety of potatoes. He is particularly interested in knowing the size of this variety, and so he selects a random sample of 30 potatoes, measures their diameters, and sends you the data.

You plot a histogram of the data, and notice that the sizes do not appear to be normally distributed, but instead look quite close to a uniform distribution.

- a) Suppose the population distribution of potato diameters was actually $U[7, 15]$ (which has mean 11 and standard deviation 2.31). What would be the (approximate) distribution of the mean of a sample of size 30 from this population?

Note: do not continue to use this population assumption for the following questions.

- b) From the farmer's sample, you calculate a mean of 10.2cm and standard deviation of 2.1cm.

Construct a 95% confidence interval for the mean diameter of the farmer's potatoes.

- c) The farmer's ideal potato size is 11.1cm. The farmer wants you to test whether or not this is the case for the new variety.

Can you reject that the hypothesis that the mean size equals 11.1cm in favour of the alternative that the mean does not equal 11.1cm at the 95% confidence level?