Economics 250 — Exercise 2 **Due 10 November 2016, in class**

Remember to include the departmental cover page. You are free to discuss the questions with classmates, but your answers should be your own.

1 Binomial investment returns [35 points]

An investor is considering the purchase of $1000 of a company's stock. On any given day, this investment has only two possible changes in value: an increase of 5% or a decrease of 4%.

The probabilities of changes in the investment's value are fixed, even if the previous days' changes are known: the investment will increase in value with probability 0.7 and decrease with probability 0.3.

Note: For this question, keep at least 4 significant digits when rounding.

a) What is the mean value (also referred to as the "expected value") of the initial $1000 investment after 1 day? After 2 days?

b) Use the binomial probability formula to find the probability of seeing at least 3 *positive* returns in the first 4 days, and find the probability of seeing at least 3 *negative* returns in the first 4 days.

c) Find the probability of having exactly two positive returns in the first four days using two different methods (show your work):

(*i*) by using your answers to *b*), and

(*ii*) by using the binomial probability formula.

d) If the investment value increases on exactly k of the four days, the value of the investment after four days will be $1000 \times 1.05^k 0.96^{4-k}$. Make a table of three rows (and leave space for a fourth row) showing:

- k, the number of days the investment value increased.
- $P(X = k)$, the probability of each value of k occurring.
- x_k, the investment value after exactly k out of four days of increasing value.

e) Use the values in your table to calculate μ_4, the expected value of the investment after 4 days.

f) Suppose that the investor will invest his money for 60 days. Find the probability of:

(*i*) seeing a positive return on at least half of the days.

(*ii*) seeing a positive return on 45 or more days.

(*iii*) seeing fewer than 18 days with negative returns.

2 Random Sampling [25 points]

Consider the distribution U$(0, 100)$, which has mean $\mu = 50$.

a) *Use Excel for this question.* Generate 20 rows of 10 values each, where each value is a draw from this distribution. For each of the 20 rows, add an 11th value which equals the mean of the first 10 values. *Tip: you can enter the Excel formula* `=AVERAGE(A1:J1)` *to calculate the mean of cells A1 through J1.*

   Plot these 20 sample means on a histogram, choosing an appropriate bin width.

b) If you repeated this process thousands of times to get thousands of sample means (each from 10 draws from U$(0, 100)$), what would you expect the histogram to look like? Where would you expect the mode to be?

c) If you repeated this process but generated samples of 20 random values instead of 10, how would you expect the histogram shape to change compared to what you described in part *b*)?

d) Suppose you have another distribution, X, for which you do not know the exact distribution, but you do know that X has a mean of 5 and standard deviation of 3. What will be the distribution of $\overline{x}$, the sample mean, for a sample constructed of 25 values drawn from X?

3 Shoe sizes [40 points]

Based on your statistics training, a shoe company hires you to perform statistical analysis to help it design and produce a new line of shoe for women. In order to proceed with the initial production, the company needs to know how many shoes to make of the various sizes. You work with the marketing department to identify the most likely customers of the new shoe and are able to conduct a simple random sample of likely customers whose feet are measured.

Your sample contains the following sizes (in centimeters):

26.1	25.9	26.2	23.3	25.0	26.2	23.3	26.1
23.5	22.4	25.0	26.1	25.7	22.3	24.9	

From this data you calculate the sample mean $\overline{x} = 24.8$ and sample standard deviation $s = 1.4467$.

a) The designer of the shoes claims that the mean female foot size equals 24.0cm. The head of the marketing department claims that the mean female foot size is *at least* 10 inches, that is, 25.4cm. For each of these two claims, write down the null and alternative hypotheses, using the usual notation.

b) Assuming that you know the true population standard deviation is $\sigma = 1.27$, for each of the two claims:

  (*i*) Calculate a test statistic that will allow you to test the statistical significance of this claim. What is the distribution of this statistic?

  (*ii*) Find the p-value associated your test statistic. Is this a one-tailed or two-tailed test?

  (*iii*) Can you reject the claim at the 95% confidence level? At the 99% confidence level? What is the highest confidence level at which you could reject the claim?

  (*iv*) If you had obtained a test statistic of *positive* 4.7 instead of what you obtained for (*i*), would you reject H_0 at the 99% confidence level?

c) Still assuming $\sigma = 1.27$, find a 97% confidence interval for the mean shoe size.

d) Now suppose that you do not know σ. For each of the two claims:

  (*i*) Calculate a test statistic that will allow you to test the statistical significance this claim. What is the distribution of this statistic? *Remember to include any relevant parameters of the distribution in your answer, if appropriate.*

  (*ii*) Can you reject the claim at the 90% confidence level? At the 95% confidence level?

e) Still supposing σ is unknown, find a 95% confidence interval for the mean shoe size.

f) Calculate the power of the test: $H_0 : \mu = 25.0, H_a : \mu \neq 25.0$ with $n = 15$ and $\sigma = 1.27$ at a significance level of $\alpha = 0.05$ when the true population mean equals 24.0cm.