

# Usage- versus cost-based pricing: lessons from the CRTC's Internet regulations

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# 1 Introduction

On 6 May 2010 the Canadian Radio-television and Telecommunications Commission (CRTC) issued Telecom Decision CRTC 2010-255: “Applications to introduce usage-based billing and other changes to Gateway Access Services” (UBB). This decision and subsequent updates in 2010 and early 2011<sup>1</sup> added to existing CRTC regulations designed to increase competition in the Canadian internet service provider (ISP) market. Prior to such regulations, ISP markets in Canada largely consisted of regional duopolies; by regulating the use and costs of use of incumbent telephone and cable infrastructure by independent ISPs it was hoped that competition would emerge. UBB was issued at the behest of incumbents who, with the increased use of the internet for high-bandwidth activities such as streaming of videos and music, had introduced what the CRTC termed “economic Internet traffic management practices” (economic ITMPs) on their own customers and sought to introduce the same on the customers of regulated independent ISPs. This economic ITMP consisted of altering offered Internet plans by imposing a menu of Internet traffic “caps” and “overage” fees for customers who exceeded these caps. Prior to UBB, access by independent customers was limited only by the speed of the Internet connection: for a fixed cost to an independent ISP, its customers could use the full capacity of their connections non-stop.<sup>2</sup> The UBB decision extended incumbents’ economic ITMPs by applying the same menu of overage fees to customers of the independent ISPs using incumbent networks, with the cost to the independent ISPs set, ultimately, at 85 percent of the incumbents’ retail rates.

The UBB decision raised significantly more public awareness than the vast majority of CRTC decisions, including an online petition with nearly half a million signatures,<sup>3</sup> and considerable media coverage.<sup>4</sup> When Canada’s Parliament signaled its intention to overrule the CRTC’s UBB decision, the CRTC withdrew UBB and began to devise a different method of pricing access.

This paper uses a simplified model of two incumbents and one independent ISP to show that, by tying the costs of access to the retail price of the incumbents rather than to the incumbents’ marginal costs of access, the CRTC made a poor choice in its UBB decision. Not only is the policy worse than could have been achieved—by using a cost-based rather than price-based approach—but it has can lead to worse outcomes than leaving a duopoly unregulated by inducing higher prices and profits while lowering consumer surplus.<sup>5</sup>

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<sup>1</sup>Telecom Decision CRTC 2010-802, Telecom Decision CRTC 2011-44

<sup>2</sup>This could be very significant in terms of total traffic; for example, the typical connection for independent ISPs using Bell’s network at the time had 5Mbps (megabits per second; 1Mbps = 122KiB/s, 1KiB = 1024B) download capacity and 800Kbps (kilobits per second) upload capacity. At maximum capacity for 24 hours per day, in one month such a connection would download approximately 1.5TiB (1TiB = 1024GiB) of data and upload approximately 240GiB. In reality, connection overhead and imperfect line conditions could reduce these numbers by a few percentage points.

<sup>3</sup>Open Media, *Stop Telecom Price Gouging*, <http://www.stopthemeter.ca>

<sup>4</sup>See, for example, CBC News, *CRTC to review internet billing decision*, <http://www.cbc.ca/news/canada/story/2011/02/03/crtc-committee.html>

<sup>5</sup>Though this is not always the case: as will be shown in section 5.2.1, under certain parameter choices the policy may increase profits by more than it decreases consumer surplus, thus resulting in a better outcome

This paper is divided into 7 sections, including this introduction. Section 2 discusses literature related to the effects of competition introduced using similar regulatory mechanisms to the Canadian system. Section 3 provides background and details on the CRTC policies and decisions that led to UBB's enactment. Section 4 sets up and described the model used for the analysis of this paper. Section 5 contains the main results of the paper by analyzing the economic results of the model under duopoly, UBB, and cost-based regulation. Section 6 is the conclusion, while section 7 proposes ways in which the model could be extended in future work.

## 2 Literature

Although economists have paid some attention to the field of modern, broadband internet access, the field, owing to its relative youth, remains an open area of study. Most of the literature deals with the consequences of various types of “open access” regulation: enabling competition by regulating costs of reaching households through incumbent infrastructure. Papers in the literature tend to concentrate on one of two aspects of this sort of regulation: first the effect on investment decisions of incumbent firms owning the physical infrastructure, and second broadband penetration and prices.

One of the most comprehensive recent works in the area of broadband internet access is the Berkman Report, published in 2010 by the Berkman Center for Internet and Society at Harvard University.<sup>6</sup> The report was written in response to a request by the United States Federal Communications Commission to look at the status of broadband internationally to help inform FCC internet policy. One of the central messages of the report is the success in various countries of open access policies, whereby a regulator mandates that incumbents make the necessary portions of the networks available to competitors in order to avoid the prohibitively high fixed cost of building duplicate network infrastructure. At the time of the report, every country in the Organization for Economic Co-operation and Development (OECD) except for the United States, Mexico, and the Slovak Republic had introduced some sort of open access regulation.

Open access occurs most commonly in the form of regulation “unbundling” local telephone services, but the report also discusses favourable results for pushing beyond unbundling to “functional separation.” Under functional separation, a regulator forces the incumbent(s) to create an independent company owning the incumbents’ physical infrastructure whose sole purpose is to invest in and sell access to telecommunications providers, including the remaining retail business of the incumbents and potential new entrants. In the United Kingdom, in which unbundling had existed but with lackluster results, the effects of the function separation introduced in late 2005 are striking:

By the end of 2008, there were 5.5 million unbundled loops [increased from about

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than duopoly in terms of total economic surplus.

<sup>6</sup>Berkman Center (2010)

200,000 at the introduction of the policy] in the UK. Prices fell by over 16% each year between 2006-2008. While the UK's competitive market did not result in the very high speeds we see in France or Japan, our analysis of prices advertised by 78 companies in the countries we review here shows that the UK companies do have among the lowest prices in the high speed (as opposed to very high speed) category of services. In our benchmarking study, the UK now has prices that are among the top quintile of performers for all tiers of service save for the very highest speeds.<sup>7</sup>

The results were similarly positive for New Zealand, which in December 2006 jumped from no open access directly to functional separation. Broadband penetration increased significantly, while the incumbent, responding to competitive pressure, made substantial new investments in high-speed fiber-optic networks throughout New Zealand's South Island.<sup>8</sup>

The literature concerning the effect on investment of open access policies, though not directly addressing the model developed in this paper, is often presented as a reason to oppose open access policies. Though various papers, both empirical and theoretical, have been written to address the issue, there has not yet emerged a consensus on even the sign of this effect, much less the magnitude. Cambini and Jiang (2009) provides an informative survey on related papers<sup>9</sup> which offer, at best, conflicting views and conclusions on how unbundling affects investment: empirical works, often using different data and different models, find conflicting results of both positive and negative effects on investment.

More theoretical approaches to understanding the connection between investment and open access policies posit both positive and negative effects. One theory warns against setting costs too low: if firms are not adequately compensated for investment (both successful and unsuccessful), they will under-invest because of the lower (and indeed negative, if rates are set below costs) returns of that investment<sup>10</sup>. A second theory suggests that, while unbundling may reduce investment, any reduction in investment must be measured against the increase in consumer welfare resulting from the more competitive environment under unbundling<sup>11</sup>. A third theory in support of unbundling is known as the "investment ladder" or "stepping-stone" investment theory: that unbundling allows entrants into a market who will, over time, graduate from offering service over incumbents' facilities to offering facilities-based competition<sup>12</sup>.

The Berkman Report expresses a complementary theory that, it claims, is little articulated in the literature but appears to fit with observations of the decisions of many OECD countries' regulators. Duplication of facilities, it is suggested, which are expensive to create, slow

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<sup>7</sup>Berkman Center (2010), p. 87.

<sup>8</sup>Berkman Center (2010), p. 87.

<sup>9</sup>It is worth noting the criticism of Cambini and Jiang (2009) raised in the Berkman Center (2010) for the authors' failing to note significant industry support behind many of the papers reviewed, all of which reached conclusions closely aligned with the interests of the sponsoring parties.

<sup>10</sup>Gayle and Weisman (2007)

<sup>11</sup>Höffler (2007)

<sup>12</sup>Cave and Vogelsang (2003)

to change, and require very long term investment decisions, serves mainly a hedge against anticompetitive behaviour by the owners or operators of existing facilities. On its own, however, “replicating the trenches and ducts, holes and poles,” meaning complete duplication of facility infrastructure, serves as a “good, but socially costly, hedge against regulatory failure.”<sup>13</sup> Thus, according to this theory (and complementing Höffler (2007)), if a regulatory environment can protect against anticompetitive behaviour by the owner or operator of the facilities, the duplicate investment is wasteful. Instead regulators should focus on fostering investment and competition where it is more socially beneficial, i.e. at the level of services and complementary investment based on a common infrastructure.

On the more pertinent issue, from the point of view of this paper, of the effects of unbundling on broadband penetration, ignoring industry-sponsored papers, the literature weakly favours the idea that unbundling does, indeed, have a positive effect on penetration. Sraer (2008) looks at loop unbundling in France, and finds significant evidence that loop unbundling leads to increased broadband penetration. Furthermore he concludes that not all of this increase can be accounted for by decreased prices alone, and suggests that differentiation of service between incumbents and entrants plays a role in consumer preferences. Höffler (2007) uses data from European countries between 2000-2004, concluding that both inter-facilities competition between telephone and cable companies and lower unbundling rates have significant, positive effects on broadband penetration. The author also calculates, using the pricing and investment information in his data, that the total welfare effect of inter-platform competition is, at best, zero, and probably somewhat negative: the welfare gains of cross-platform competition are not, according to the author’s calculation, socially worth the cost of platform duplication.

Economics literature without industry sponsorship that deals primarily with broadband regulation in Canada is virtually non-existent. A good example of incumbent-sponsored literature is in a conference paper authored by two senior Bell executives, Krause and Bibic (2012), which brutally attacks the existence of any regulation whatsoever on the grounds that the extent of competition in Canada is so great that no regulation is required at all. An opposing view is found in Geist (2011), which provides a review of UBB and analysis of the regulatory failures that led to its enactment.<sup>14</sup> Geist concludes that “[t]he lack of competitiveness of Canadian Internet access is the product of ten years of policy neglect,” and proposes various overhauls in regulatory approach, most significantly that the CRTC should take a stronger role in the prevention of incumbent anti-competitive abusive behaviour to foster stronger competition in the Canadian ISP market.

A more economically oriented—and independent—analysis is to be found by returning to the Berkman Report, which includes a section dedicated to the situation in Canada as of the report’s publication<sup>15</sup>. It portrays Canada as a country that started strongly in the early years of broadband development but has lagged behind in more recent years in penetration,

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<sup>13</sup>Berkman Center (2010), p. 93

<sup>14</sup>It must be noted, however, that Geist’s paper is also sponsored by industry sponsorship: though his basic ideas originated in unsponsored blog posts, the full report was, as Geist discloses, sponsored by Netflix, an internet company with strong financial motives in opposition to that of the incumbent ISPs.

<sup>15</sup>Berkman Center (2010), section B, pp. 247-257.



speed, and price. The report attributes this as most likely the result of a poor unbundling implementation: “Canada in particular offers an example of halfhearted efforts to impose unbundling, and increasingly heavy reliance on competition between local telephone and cable incumbents. Its results, as our benchmarking study shows, have been weaker than those of other countries we review here.”<sup>16</sup> The report’s comparisons to other OECD countries place Canada as a “weak to mid-pack performer” in speed, falling in the fourth or fifth quintiles, with prices generally falling in the third or fourth quintile<sup>17</sup>

The Berkman Report makes particular note of Telecom Order CRTC 2009-484 (discussed more extensively in section 3), which allowed incumbents to impose bandwidth caps on competitors using an incumbents facilities.<sup>18</sup> The report also draws attention to the CRTC’s approach, unique among OECD countries, of setting unbundling prices at long-run incremental costs plus a 25% markup (reduced to 15% in 2002), suggesting that this markup may be responsible for Canada having the highest monthly rates for unbundled local loop access among OECD countries.<sup>19</sup> In conclusion, the report paints a dismal picture of broadband in Canada:

In the area of competition policy, Canada implemented unbundling rules formally in 1997, but its regulated rates were high relative to the rest of the OECD, and it consistently imposed sunsets on all or some category of regulation. As a practical matter, its market has evolved toward a regional market with relatively low investments in other regions by incumbents prominent in one region. Most competition in any given region is between the telephone and cable company that was locally dominant in the past. ... Canada continues to see itself as a high performer in broadband, as it was early in the [2000–2009] decade, but current benchmarks suggest that this is no longer a realistic picture of its comparative performance on several relevant measures.<sup>20</sup>

Some papers, particularly those written during the CRTC’s consultations regarding UBB and its replacement, have focussed on the costs of network transit and internet connectivity, which are core components of the model in this paper. Geist (2011) includes an appendix that attempts to estimate the internal traffic costs using information from public records of CRTC submissions from incumbent ISP Bell Canada, relying on estimates to fill in the redacted figures in the public records. He estimates a long run marginal cost, including operating and investment costs, of approximately \$0.07/GB for the cost of internal network traffic, significantly below the \$1/GB or \$1.50/GB retail rates for overage set by Bell for its retail customers. Other studies—though similarly sponsored or commissioned by third parties—vary in specific estimates but generally agree that the actual costs are significantly below the retail prices set by Bell and other incumbents. A report submitted by Sandvine

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<sup>16</sup>Berkman Center (2010), p. 248.

<sup>17</sup>Berkman Center (2010), p. 248. In terms of speed, the first quintile contains the countries with the fastest internet, while on price the first quintile consists of countries with the lowest cost of internet.

<sup>18</sup>Berkman Center (2010), p. 248.

<sup>19</sup>Berkman Center (2010), p. 255.

<sup>20</sup>Berkman Center (2010), p. 257.

Incorporated to the CRTC in response to Telecom Notice of Consultation CRTC 2011-77, for example, estimates costs at \$0.17, though this figure includes the cost from household to internet, which suggests the internal network transit costs are somewhat lower than \$0.17.<sup>21</sup> Lemay-Yates Associates, Inc. (2011b), commissioned by Netflix, estimates costs per gigabyte for internal network traffic at \$0.01, with an upper limit of \$0.02 per GB for service in lightly trafficked network areas.

The model developed in this paper, while applied to the Canadian telecommunications environment created by the CRTC, is not inherently specific to that environment but rather concerns the pricing behaviour of firms under the regulatory structures enacted by the CRTC. The basic design of the model (see section 4) loosely follows the linear-demand, differentiated-product, Bertrand, monopolistic competition model of Singh and Vives (1984)—itself an adaptation of Dixit (1979)—but adapted for three firms with differing cost structures. Though Singh and Vives (1984) compares welfare differences between Bertrand and Cournot competition for differentiated products, the model and intuition of parameters provides a useful starting point for welfare analysis of a similar model.

## 3 Usage-based billing (UBB)

### 3.1 Precursor to UBB

In a series of decisions in the late 1990s and early 2000s, the CRTC established rules and tariffs that enabled independent internet service providers to offer internet access through the network infrastructure of incumbent internet providers. The rationale of the regulation was that it would reduce the market power of the incumbents, which generally operated as a duopoly of one “incumbent local exchange carrier” (ILEC)<sup>22</sup> and one incumbent cable carrier<sup>23</sup> in most regions of Canada, with essentially no competition between multiple ILECs or cable carriers in any given area.<sup>24</sup> The CRTC’s regulations were introduced separately for ILECs and cable carriers, with ILECs mostly covered under regulation known as residential “Gateway Access Service” (GAS), while cable companies are covered under residential “third-party Internet access” (TPIA) service regulations. For the purpose of analysis in this paper, the specific differences between GAS and TPIA regulations—which mainly consist of technical differences due to the different technologies employed by ILECs and cable carriers—are not important.<sup>25</sup> By enabling wholesale access at regulated prices, the CRTC

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<sup>21</sup>The report itself is not publically available, however is discussed in detail in Lemay-Yates Associates, Inc. (2011a), submitted to the CRTC on behalf of Netflix, written in response to the Sandvine report and other CRTC submissions.

<sup>22</sup>Less officially known as local telephone operators. For the vast majority of Canadians, this means one of Bell Aliant, Bell Canada, MTS Allstream, SaskTel, and TELUS, roughly in order from east to west in terms of service areas.

<sup>23</sup>The most significant being EastLink, Quebecor/Videotron, Cogeco, Rogers, and Shaw, again roughly ordered by service area from east to west.

<sup>24</sup>Berkman Center (2010), p. 249.

<sup>25</sup>Indeed in the UBB and later, related decisions, the CRTC treats GAS and TPIA equally.

hoped to attract new, competitive ISPs to the market with the hope that the increased competition would reduce prices and spur innovation.

As internet usage exploded, the incumbents attempted to discourage heavy use of their networks through two means: first, by slowing down services, such as peer-to-peer file sharing applications, known to consume substantial amounts of bandwidth; and second, by adding usage caps to existing internet contracts.

The first step towards UBB occurred on 21 December 2006 when the CRTC issued Telecom Decision CRTC 2006-77. Though this decision mainly addressed various TPIA costs and technical details between incumbent carriers and TPIA providers, it also addressed the issue of high-usage users, ruling that it “considers it appropriate that each cable carrier be provided the ability to manage the potential negative outcome of high-consuming bandwidth end-users in a manner that does not degrade the Q of S [quality of service] to all end-users, whether it is the cable carrier’s end-user or the competitor’s end-user.” The Commission added the proviso that the application of any such management must, however, apply “equal treatment” to incumbent and TPIA customers.<sup>26</sup>

On 12 August 2009 in Telecom Order CRTC 2009-484 the CRTC approved, on an interim basis, Bell’s petition to create a precursor to UBB. In this early UBB decision, the CRTC approved a overage fee of \$0.75 per GB over 300GB used by a wholesale customer in a month, and since this mirrored Bell’s charges to its own customers \$1 per GB over 300GB, this satisfied the “equal treatment” requirement that had been established in Telecom Decision CRTC 2006-77. Initially the CRTC allotted 90 days between the time Bell imposed the fee on its own customers to the date independent ISPs would be subject to the UBB fees, though Telecom Decision CRTC 2009-658 postponed this date to a later, separate decision.

At the same time as postponing implementation, the CRTC issued a policy on ITMPs, Telecom Regulatory Policy CRTC 2009-657, that clarified when ITMPs are acceptable. The Commission also encouraged the use of economic ITMPs over technological ITMPs as a means of managing ISP traffic

as they link rates for Internet service to end-user consumption. Economic ITMPs also provide greater transparency to users than technical ITMPs, as they are reflected in monthly bills. Furthermore, these practices match consumer usage with willingness to pay, thus putting users in control and allowing market forces to work.<sup>27</sup>

The policy formed the last piece of the foundation upon which UBB would be built.

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<sup>26</sup>Telecom Decision CRTC 2006-77, paragraph 248.

<sup>27</sup>Telecom Regulatory Policy CRTC 2009-657, paragraph 40.

## 3.2 UBB implementation

Armed with approval of the the early form of UBB and an indication of pre-approval of “economic ITMPs”, Bell reacted by proposing usage caps beginning at 60GB for its own customers, coupled with an application to the CRTC to impose those same caps on independent ISPs using Bell’s network. Usage over 60GB by an independent ISP’s customer would be billed to the independent ISP at \$1.125 per GB between 60 and 300GB, to a maximum monthly charge of \$22.50 (incurred for any level of usage between 80GB and 300GB)—each the 25% discounted value of Bell’s retail fees of \$1.50 per GB over 60GB with a \$30 maximum. Any usage above 300GB would maintain the \$0.75 per GB fee established in the earlier UBB decision. Furthermore, Bell offered its own customers a so-called “insurance plan”<sup>28</sup> for \$5/month that increased the basic cap by 40GB, from 60 to 100GB (with overage fees then applying between 100 and 120GB). Though Bell presented this as an economic ITMP aimed at discouraging excess use, the structure seems inconsistent with such a purpose: it imposed upon end-users a zero marginal cost of internet use between 0 and 60GB (or 100GB with the “insurance plan”), and between 80 (or 120) and 300GB, while imposing a higher marginal cost for use between 60 and 80GB than for use above 300GB.

The CRTC largely approved Bell’s proposal on 6 May 2010 in Telecom Decision CRTC 2010-255. Because “rates vary with the amount of usage generated by each end-user of a GAS ISP,” the CRTC agreed that Bell’s proposal “would incent heavy end-users to reduce their usage” and therefore qualified for as an economic ITMP.<sup>29</sup> The CRTC did, however, institute some relatively minor changes: it mandated that the “insurance” Bell offered its own customers must also be offered to the independent ISPs, with the same 25% discount applied to the other fees; it furthermore required that Bell impose the UBB rates on all of its own customers (including those on plans without usage caps, last offered by Bell in 2006) before UBB could be imposed on the customers of the independent ISPs.

Soon after the Telecom Decision CRTC 2010-255, Bell requested that the CRTC reconsider, claiming “that there is substantial doubt as to the correctness of Decision 2010-255” and that “[t]he Commission erred in fact and law” in various aspects of the decision.<sup>30</sup> The CRTC responded by amending the decision in Telecom Decision CRTC 2010-802, released 28 October 2010. In it the CRTC rescinded the requirement that Bell impose UBB on all of its retail customers, instead adding an exception for independent ISP customers whose had established their current service in 2006 or earlier. Additionally Bell argued that discounting overage rates by 25% was not equivalent treatment, and was furthermore not fair because usage-based components in the pricing of TPIA providers were not discounted by 25%, thus regulatory symmetry was not achieved. The CRTC agreed, temporarily removing the 25% discount on overage fees while also announcing that it would deal with the appropriate level of discount, if any, in a later decision. Less than 2 months after the amended decision, Bell responded by lowering its traffic caps for its Ontario customers from 60GB to 25GB,

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<sup>28</sup>The notion of insurance offered against additional fees imposed by the insurer itself seems, at best, a dubious definition of insurance.

<sup>29</sup>Telecom Decision CRTC 2010-255, paragraph 21

<sup>30</sup>Bell Aliant Regional Communications, Limited Partnership and Bell Canada (the Companies), 2010

increasing its UBB rates, and petitioning the CRTC to approve the new usage caps and fees for its independent ISP customers as well.<sup>31</sup>

Meanwhile, in its consultation to determine an appropriate discount for UBB fees, the CRTC received various inputs from interested parties. The inputs divided along predictable lines: most of the incumbents<sup>32</sup> took the position that UBB rates should not be discounted at all, because any discount in fees would lead to different incentives for incumbent customers and independent ISP customers. The independent ISPs took the position that a significant discount should be present to allow for a profit margin and to allow increased service differentiation, without which the independent ISPs would be unable to compete through pricing or differentiation from the offerings of the incumbents. They also made an economic argument that, since UBB was not cost-based, it amounted to an anti-competitive subsidy from independents to incumbents. Reducing the usage fees, they argued, would therefore reduce this anti-competitive subsidy. The CRTC released its decision in Telecom Decision CRTC 2011-44 on 25 January 2011: a discount was necessary because otherwise the independent ISPs would not be meaningfully able to differentiate their services from the incumbents' services, but the discount could not be large because that would weaken the efficacy of UBB as the economic ITMP that the CRTC had already approved in Telecom Decision CRTC 2010-255. Based on these two findings, the Commission decreed that 15% was the appropriate discount. No discussion refuting the claim of an anti-competitive subsidy nor any attempt to justify such a subsidy was offered in the decision.

### 3.3 Replacement of UBB

This final decision caught the attention of the media and many Canadians in a way few prior CRTC decisions had done. An online poll amassed almost half a million signatures opposed to UBB, and the lead Commissioner was called before Parliament to explain the CRTC's justifications for the UBB decision.<sup>33</sup> Finally, as the Canadian Parliament signalled its intention to overrule the UBB decision, the CRTC "voluntarily" withdrew the decision "of its own initiative,"<sup>34</sup> initiating a consultation to devise an alternative to UBB for wholesale billing practices.<sup>35</sup>

That alternative came in Telecom Regulatory Policy CRTC 2011-703 which advocated an entirely different approach to billing independent ISPs. In it, independent ISPs would pay a fixed fee per customer, with an additional fee for the maximum aggregate bandwidth capacity of its customers, sold in 100Mbps increments. The idea was that, because high-traffic users would incur significant use of its available capacity, independent ISPs would then be free to impose their own ITMPs, economic or otherwise, to manage congestion issues appropriately.

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<sup>31</sup>Telecom Regulatory Policy CRTC 2011-703, paragraph 13.

<sup>32</sup>Only MTS Allstream, the ILEC serving Manitoba, sided with the independent ISPs.

<sup>33</sup>CBC News, 2011.

<sup>34</sup>Telecom Notice of Consultation CRTC 2011-77, paragraph 11.

<sup>35</sup>Telecom Notice of Consultation CRTC 2011-77.

In deciding the appropriate rates, the CRTC employed a cost-based approach, using cost data submitted by the incumbent ISPs, plus a markup for future network investment. In its design, the aggregated approach offered a considerably more competition-friendly approach, while offloading the responsibility and impact of network congestion onto the independent ISPs. Some evidence, however, suggests that the CRTC erred by using cost data submitted by the incumbent ISPs rather than using an independent process to determine costs. In the decision, approved costs per 100Mbps capacity increments vary widely from \$281 (MTS Allstream) to \$2213 (Bell) among ILECs, and \$1251 (Rogers) to \$2695 (Cogeco) among cable incumbents.<sup>36</sup> Since technologies used by difference ILECs are similar, as are those of the cable incumbents, it appears that the CRTC did not attempt to resolve the obvious discrepancy in costs, instead merely accepting the costs submitted by each firm. With approved costs differing by a factor of nearly 10 across incumbent ISPs and no justification begin given in the decision for the cost discrepancies, the CRTC's claims of a cost-based approval are dubious.

### 3.4 Analysis of UBB decisions

One finds in the CRTC UBB decisions a pattern of internally consistent but ultimately unsound rulings. For example, once approval was granted for economic ITMPs—which seems appropriate in principle—subsequent filings claiming justification for UBB as an “economic ITMP” were approved as such, on the basis that the UBB fees do indeed reduce usage. No regard appears to have been given to whether the proposed rates were *solely* economic ITMPs or also an extension of market power. Once some part of UBB was determined to be an economic ITMP, the CRTC treated UBB as if it was *entirely* an economic ITMP. Nor, once it became apparent how the incumbents would use the regulation in practice, did the Commission undertake any retroactive analysis of the earlier approval of economic ITMPs.

Such analysis through absolute categories (for example: since economic ITMPs can be justified, UBB at least partly achieves what an economic ITMP should achieve, therefore UBB must be a justifiable economic ITMP) rather than analysis of likely results of its decisions, kept the CRTC consistent within its own earlier decisions but ultimately led to an unsound result with significant public and political opposition. Indeed, through the UBB affair, the decisions of the CRTC seem more concerned with following earlier decisions than with analysis of economic outcomes such as competitive effects, pricing, or the welfare of consumers. Most striking, and most indefensible, is that the CRTC offered little economic justification in its decisions. Whether that is from lack of or disregard of economic advice is unclear, yet its rulings particularly around the UBB decision deal directly with market power and competition, issues firmly within the realm of economics. In short, the CRTC regarding UBB made economic decisions without taking appropriate consideration of the economic effects of those decisions.

Though the CRTC ultimately overturned and replaced UBB with a superior system better

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<sup>36</sup>Telecom Regulatory Policy CRTC 2011-703, Appendix 1: “Approved rates for wholesale residential high-speed access services”

able to align the incentives of users with the incentives of their suppliers, it is evident that it only did so as a result of the Canadian Parliament’s threat to reverse the decision. Moreover, even though the UBB replacement was significantly better than UBB, the CRTC still failed in that ruling by accepting costs with widely differing values across different firms, without independent analysis of appropriate costs, or any attempt to induce truthful cost reporting by incumbents’ submissions to the CRTC.

This paper analyses usage-based billing in a simplified theoretical model and shows how, in this framework, will be always socially inferior to a cost-based approach, and moreover that it can potentially result in a socially worse outcome than leaving the incumbents to operate as a duopoly.

## 4 The Model

### 4.1 Overview

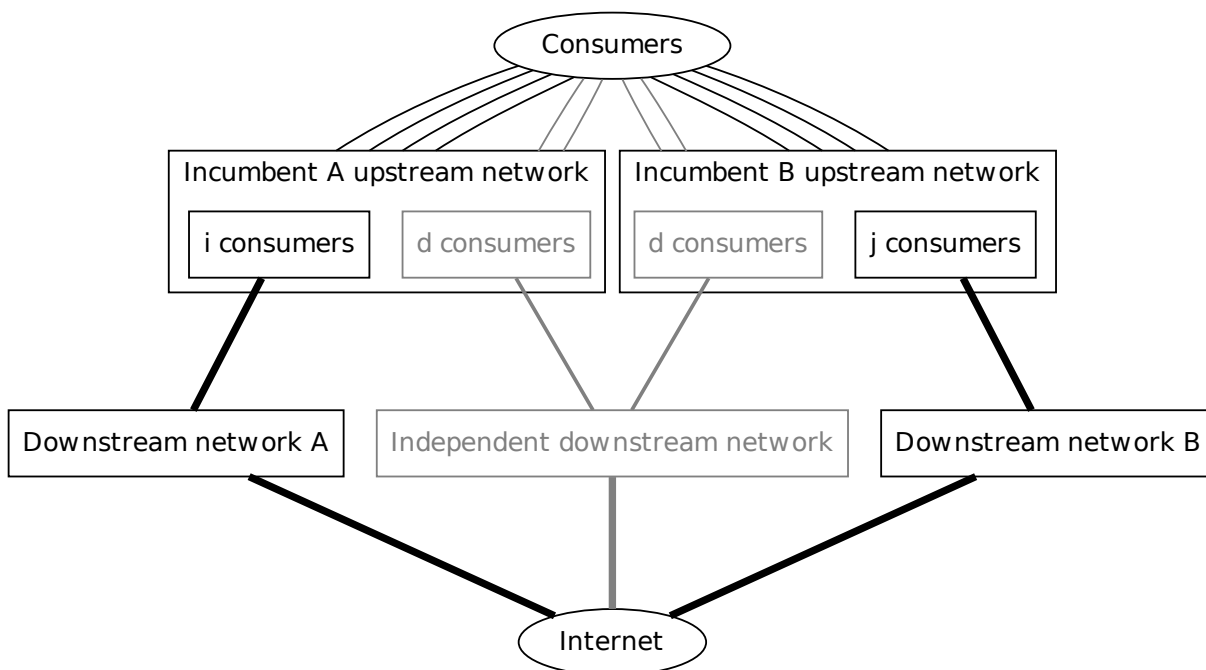
The ISP market in this paper is modelled as one with three potential firms: two “upstream” firms, operating as price-setting competitors, and a “downstream” firm, also a price-setter, but whose service requires the use of a portion of the upstream firms’ networks. Services offered by the three firms are imperfect substitutes, following the empirical evidence of Sraer (2008), but are identically demanded; that is, each firm’s demand curve is assumed symmetric to that of the other two firms.

Three regulatory cases are investigated using this model, following the regulatory structures imposed by the CRTC. Initially, there is no regulated access at all, thus barring the downstream firm from entering, resulting in the upstream firms operating as duopolists. The second case has the firms operating under a UBB-like regulation, where the regulator bases the downstream firm’s access costs on the upstream firm’s retail price. The final case has the firms operating under a regulation that sets the access cost as a fixed amount based on the actual cost incurred by the upstream firms.

The term “upstream network” is used throughout this paper to refer to the portion of the network owned by the upstream firms, consisting of the physical connections from the incumbents’ office(s) to individual households, while the “downstream network” refers to the portion of the network at which traffic has been aggregated and is routed to the internet by each of the three firms through their own third-party internet connection arrangements. This structure is depicted in the following diagram, illustrating the connection path from household to internet under regulation in which all three firms participate. Black lines represent upstream firm customer traffic, while gray lines depict downstream firm customer traffic. The thin lines indicate individual consumer traffic, while thicker lines indicate aggregated traffic.<sup>37</sup>

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<sup>37</sup>This depiction is essentially the same as that found in related works when depicting the Canadian internet situation, e.g. Lemay-Yates Associates, Inc. 2011b.



## 4.2 Assumptions

The model contains various simplifying assumptions for tractability, while leaving the core mechanism of UBB intact.

Each of the three goods is a compound good representing both quantity (i.e. number of connections) and quality (i.e. connection speed, traffic use) of connectivity—in other words, a simplified version of usage. In reality firms offer a menu of choices, including per-connection fees, traffic caps, overage fees, and overage “insurance” fees, each of which may differ across multiple offered speed tiers. Moreover many ISPs offer lower introductory rates, discounts for ordering multiple services together, or various promotional offers. All of these pricing mechanisms contribute significant complexity while not contributing to the analysis of UBB; thus the model uses a single, simplified quantity for each firm representing usage (and single associated price). A larger quantity thus simply indicates more use of the firm’s network, which could be interpreted as more users, more Internet use by existing users, or both. This assumption also means that, from the point of view of both consumer utility and firms, a given quantity of customers with a particular amount of usage is exactly equivalent to twice the number of customers each with half as much usage.

Firms are assumed to have negligible fixed costs, while marginal cost to the firms is broken into two components: an “upstream” cost component, incurred by an upstream firm for usage by both its own customers and the customers of downstream firm using its network; and a “downstream” cost that is incurred by each of the three firms for its own customers. It is assumed that the upstream cost is identical for both upstream firms and the downstream



cost identical for all three firms. It is assumed that downstream traffic imposes no additional “interconnection” cost compared to the cost of an upstream firm’s own customer traffic—in other words, that a unit of the good costs, in aggregate, the same amount whether provided by a downstream or upstream firm.

Both upstream and downstream marginal costs are assumed to include appropriate amounts for long-run network upgrades, maintenance, and investment, which are assumed to scale linearly with the quantity of service provided (thus maintaining constant marginal costs). Upstream costs should, as discussed in section 2, ideally include not only actual investment, but also appropriate unsuccessful investment initiatives, so as to properly price the risk associated with investment.

It is also assumed that downstream access can only occur at the regulator-defined price; in other words that lower prices cannot be negotiated independently by firms. Additionally, the downstream firm is assumed to buy access from each upstream firm in equal proportions. These two assumptions simplify the model by eliminating the necessity of establishing a second market between the downstream and upstream firms for downstream firm access.

In the model, firms set prices simultaneously and are able to satisfy their demand at any given price. Firms always enter the market when doing so provides positive profits.

Consumers are assumed to be price-takers.

### 4.3 Notation

The two upstream firms will be denoted with subscripts  $i$  and  $j$ ; the downstream firm by with subscript  $d$ . The subscript  $u$  is used for notational convenience to refer to upstream values when the two upstream firm values are identical.

The marginal costs involved are denoted  $c_u$ , for costs incurred by an upstream firm for all carried upstream firm and downstream firm traffic; and  $c_d$  for each firm’s downstream traffic costs.  $c_{r,i}$  and  $c_{r,j}$  (simplified to  $c_r$  when equal) denote the regulated prices at which the downstream firm is able to buy the required upstream access component from the upstream firms for its own customers. These values need not be constant: in the UBB model they will be functions of each upstream firm’s price.

### 4.4 Consumers

Consumers have quasilinear utility from the three goods, and have an available outside, competitive, numeraire good,  $n$ . Consumer utility is given by the quadratic utility function:

$$U(q_i, q_j, q_d, n) = n + \alpha(q_i + q_j + q_d) - \frac{\beta}{2}(q_i^2 + q_j^2 + q_d^2) - \gamma(q_i q_j + q_i q_d + q_j q_d) \quad (4.4.1)$$

where  $q_i$ ,  $q_j$ ,  $q_d$ , and  $n$  are the choice variables of the consumer for quantities purchased from the three firms, and numeraire quantity, respectively.  $\alpha$ ,  $\beta$ , and  $\gamma$  are positive constants. Two assumptions are made regarding these variables:

$$\alpha > c_u + c_d \quad (4.4.2)$$

$$\beta > \gamma > 0 \quad (4.4.3)$$

Assumption (4.4.2) is necessary to ensure that any market exists at all: it simply states that, for each good, the maximum willingness-to-pay of the good (which occurs when  $q_i = q_j = q_d = 0$ ) exceeds the marginal cost of producing that good. Without this assumption, no firm would enter the market.

The second assumption, (4.4.3), ensures that the price of a good is affected more strongly by changes in the quantity of the same good than by changes in the quantity of a competing good. This relationship will become more apparent in the inverse demand functions that follow ((4.4.5)–(4.4.7)). This assumption also ensures that the utility function is concave. The second part of the assumption,  $\gamma > 0$ , is simply an assumption that the goods are substitutes rather than independent goods ( $\gamma = 0$ ) or complements ( $\gamma < 0$ ).

The relationship between  $\beta$  and  $\gamma$  in this model is worth considering. In particular,  $\frac{\gamma}{\beta} \in (0, 1)$  constitutes a measure of product differentiation, as per Singh and Vives (1984). As this ratio approaches 0, the goods become independent; as it approaches 1, the goods become perfect substitutes. Imposing  $\beta > \gamma > 0$  is thus specifying that the goods are substitutes, but not perfect ones. As will be shown later in the resulting demand equations,  $\beta > \gamma$  also ensures that aggregate quantity decreases with increases in one (or more) prices, *ceteris paribus*.

It can be reasonably expected that in the market for internet service providers, which this paper is primarily modelling,  $\frac{\gamma}{\beta}$  is relatively close to 1, although no such assumption is made in the analysis in this paper.

The associated Lagrangian for the consumer maximization problem with income  $w$  is thus:

$$\mathcal{L} = U(q_i, q_j, q_d, n) - \lambda(p_i q_i + p_j q_j + p_d q_d + n - w) \quad (4.4.4)$$

The first-order condition for the numeraire yields  $\lambda = 1$ ; the three first-order conditions for the  $q$  variables then yields the inverse demand functions for the three firms:

$$p_i = \alpha - \beta q_i - \gamma(q_j + q_d) \quad (4.4.5)$$

$$p_j = \alpha - \beta q_j - \gamma(q_i + q_d) \quad (4.4.6)$$

$$p_d = \alpha - \beta q_d - \gamma(q_i + q_j) \quad (4.4.7)$$

Solving these three equations for  $q_i, q_j, q_d$  yields the demand functions of the three firms:

$$q_i(p_i, p_j, p_d) = \frac{\alpha}{\beta + 2\gamma} - \frac{\beta + \gamma}{(\beta + 2\gamma)(\beta - \gamma)} p_i + \frac{\gamma}{(\beta + 2\gamma)(\beta - \gamma)} (p_j + p_d) \quad (4.4.8)$$

$$q_j(p_i, p_j, p_d) = \frac{\alpha}{\beta + 2\gamma} - \frac{\beta + \gamma}{(\beta + 2\gamma)(\beta - \gamma)} p_j + \frac{\gamma}{(\beta + 2\gamma)(\beta - \gamma)} (p_i + p_d) \quad (4.4.9)$$

$$q_d(p_i, p_j, p_d) = \frac{\alpha}{\beta + 2\gamma} - \frac{\beta + \gamma}{(\beta + 2\gamma)(\beta - \gamma)} p_d + \frac{\gamma}{(\beta + 2\gamma)(\beta - \gamma)} (p_i + p_j) \quad (4.4.10)$$

For illustrative simplification, the system can be reparameterized using:

$$A \equiv \frac{\alpha}{\beta + 2\gamma} \quad B \equiv \frac{\beta + \gamma}{(\beta + 2\gamma)(\beta - \gamma)} \quad \Gamma \equiv \frac{\gamma}{(\beta + 2\gamma)(\beta - \gamma)} \quad (4.4.11)$$

which yields the reparameterized demand system:

$$\begin{aligned} q_i(p_i, p_j, p_d) &= A - Bp_i + \Gamma(p_j + p_d) \\ q_j(p_i, p_j, p_d) &= A - Bp_j + \Gamma(p_i + p_d) \\ q_d(p_i, p_j, p_d) &= A - Bp_d + \Gamma(p_i + p_j) \end{aligned} \quad (4.4.12)$$

Note that, since  $\alpha, \beta,$  and  $\gamma$  are positive, and  $\beta > \gamma$  (by (4.4.3)), these new parameters are also positive.

Assumption (4.4.3),  $\beta > \gamma$ , implies  $\beta + \gamma > 2\gamma$ , which in turn implies  $B > 2\Gamma$ , which, as is apparent here, lends support to the assumption itself: an increase in any one price increases the quantities of the other firms by less than it decreases the quantity of the firm raising its price. In other words, such a price increase decreases *aggregate* quantity demanded.

Substitutability in the new parameter space is captured by  $\frac{\Gamma}{B} \equiv \frac{\gamma}{\beta + \gamma}$ , thus  $\frac{\gamma}{\beta} \in (0, 1)$  implies  $\frac{\Gamma}{B} \in (0, \frac{1}{2})$ , where 0 corresponds to independent goods, and  $\frac{1}{2}$  to perfect substitutes. This relationship between  $\frac{\gamma}{\beta}$  and  $\frac{\Gamma}{B}$  is positive but non-linear. Noting that  $\frac{\Gamma}{B} = \frac{\gamma}{\beta + \gamma}$ , this substitutability parameter offers an alternative measure of substitutability to that discussed above:

$$\Sigma \left( \frac{\gamma}{\beta} \right) \equiv 2 \frac{\Gamma}{B} = \frac{2}{1 + \left( \frac{\gamma}{\beta} \right)^{-1}} \quad (4.4.13)$$

For any ratio  $\frac{\gamma}{\beta} \in (0, 1)$ ,  $\Sigma \left( \frac{\gamma}{\beta} \right) \in (0, 1)$  thus provides the ratio of the increase in demand for a price reduction of one firm to the increase in demand of an increase of the same price in each of the firm's competitors. Thus  $\Sigma = \frac{1}{2}$  indicates a market where quantity demanded of a firm increases twice as much for a price decrease of that firm as it would for a price increase of the same magnitude of both competitors.<sup>38</sup>

<sup>38</sup> Though  $\frac{\gamma}{\beta}$  are reported in section 5 rather than  $\Sigma \left( \frac{\gamma}{\beta} \right)$  values, the following table offers a few numeric values of  $\Sigma \left( \frac{\gamma}{\beta} \right)$  that can provide a rough guide on the relationship between the two measures.

$\frac{\gamma}{\beta}$	0	0.1	0.2	0.3333	0.5	0.6	0.6667	0.8	0.9	1
$\Sigma \left( \frac{\gamma}{\beta} \right)$	0	0.1818	0.3333	0.5	0.6667	0.75	0.8	0.8889	0.9474	1

This reparameterization is only for illustrative purposes and will not be used in the versions of the model that follow in section 5; while it simplifies the system as displayed above, using it in the later model variations would complicate rather than simplify the mathematical expressions involved.

## 4.5 Firms

Firms set prices to maximize their respective profits, given by:

$$\pi_i = (p_i - c_u - c_d)q_i(\cdot) + (c_r - c_d)\frac{q_d(\cdot)}{2} \quad (4.5.1)$$

$$\pi_j = (p_j - c_u - c_d)q_j(\cdot) + (c_r - c_d)\frac{q_d(\cdot)}{2} \quad (4.5.2)$$

$$\pi_d = (p_d - c_r - c_d)q_d(\cdot) \quad (4.5.3)$$

where  $q_i(\cdot)$ ,  $q_j(\cdot)$ , and  $q_d(\cdot)$  are as given in equations (4.4.8)–(4.4.10).

## 4.6 Methodology

Many of the results that follow have been calculated by hand and verified computationally, using version 15.01 of the Maple software package. In some cases, where numerical results are used (for example, in determining whether and in what range of  $\frac{\gamma}{\beta}$  various coefficients are positive or negative, beginning in section 5.2.1), the resulting numerical values of roots and asymptotes are determined entirely using Maple’s numerical techniques.

The Maple Worksheet (.mw) files associated with these calculations and PDF versions of those worksheets, including resulting calculations, are available at <https://imaginary.ca/papers/ubb>.

# 5 The Model Explored

## 5.1 Duopoly: No downstream competitor

In order to maintain an equivalent demand function between the market with and without the downstream competitor, the case of no downstream competitor is considered as if the downstream competitor’s price was set to its “choke” price: the price at which its quantity demanded reaches zero.<sup>39</sup> The choke price for the downstream firm is given by  $q_d(p_i, p_j, p_d) =$

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<sup>39</sup>Alternatively, the equations can be derived by starting from the utility function with  $q_d = 0$ , i.e.

$$U(q_i, q_j, 0, n) = n + \alpha(q_i + q_j) - \frac{\beta}{2}(q_i^2 + q_j^2) - \gamma q_i q_j$$

0, which from (4.4.10) is:

$$p_d^{\text{choke}} = \frac{(\beta - \gamma)\alpha + \gamma(p_i + p_j)}{\beta + \gamma} \quad (5.1.1)$$

Substituting this into the upstream firms' demand functions from (4.4.8) yields:<sup>40</sup>

$$q_i^{\text{duo}}(p_i, p_j) = \frac{(\beta - \gamma)\alpha - \beta p_i + \gamma p_j}{(\beta + \gamma)(\beta - \gamma)} \quad (5.1.2)$$

Profits for upstream firm  $i$  are:

$$\pi_i^{\text{duo}} = (p_i - c_u - c_d)q_i^{\text{duo}}(\cdot) \quad (5.1.3)$$

Resulting equilibrium prices and quantities are:

$$p_i^{\text{duo}} = p_j^{\text{duo}} = \frac{(\beta - \gamma)\alpha + \beta(c_u + c_d)}{2\beta - \gamma} \quad (5.1.4)$$

$$q_i^{\text{duo}} = q_j^{\text{duo}} = \frac{\beta(\alpha + c_u - c_d)}{(\beta + \gamma)(2\beta - \gamma)} \quad (5.1.5)$$

Resulting consumer surplus and firm profits, from (4.4.1) and (5.1.3) respectively, are:<sup>41</sup>

$$CS^{\text{duo}} = \frac{\beta^2(\alpha - c_u - c_d)^2}{(2\beta - \gamma)^2(\beta + \gamma)} \quad (5.1.6)$$

$$\pi_i^{\text{duo}} = \pi_j^{\text{duo}} = \frac{\beta(\beta - \gamma)(\alpha - c_u - c_d)^2}{(2\beta - \gamma)^2(\beta + \gamma)} \quad (5.1.7)$$

and thus total surplus is:

$$\begin{aligned} TS^{\text{duo}} &= CS^{\text{duo}} + 2\pi^{\text{duo}} \\ &= \frac{\beta(3\beta - 2\gamma)}{(2\beta - \gamma)^2(\beta + \gamma)}(\alpha - c_u - c_d)^2 \end{aligned} \quad (5.1.8)$$

These results, while not particularly profound on their own, form a basis of comparison with the following model variations.

## 5.2 Price-based regulation (UBB)

UBB in this model is introduced by setting the regulated cost for the upstream portion of access set as a multiple  $\eta$  of the upstream firms' retail prices, i.e.  $c_{r,i} = \eta p_i$  and  $c_{r,j} = \eta p_j$ . It is assumed that there is sufficient demand for the downstream firm to earn positive profits at equilibrium.<sup>42</sup>

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The resulting demand equations are, of course, identical.

<sup>40</sup>For this section, only functions for firm  $i$  are shown; functions for firm  $j$  are symmetrically identical.

<sup>41</sup>Utility is simply (4.4.1) evaluated at the equilibrium values:  $U(q^{\text{duo}}, q^{\text{duo}}, 0, w - 2p^{\text{duo}}q^{\text{duo}})$ . Consumer surplus, taking into advantage the quasilinearity of utility, is thus simply  $U(\cdot) - w = U(q^{\text{duo}}, q^{\text{duo}}, 0, -2p^{\text{duo}}q^{\text{duo}})$ .

<sup>42</sup>Algebraically,  $\eta p_u + c_d < p_d^{\text{choke}}$ ,  $u \in \{i, j\}$  must be satisfied at equilibrium, with  $p_d^{\text{choke}}$  given by (5.1.1).

In this scenario, the firms' profit functions are:

$$\pi_i = (p_i - c_u - c_d)q_i(\cdot) + (\eta p_i - c_u)\frac{q_d(\cdot)}{2} \quad (5.2.1)$$

$$\pi_j = (p_j - c_u - c_d)q_j(\cdot) + (\eta p_j - c_u)\frac{q_d(\cdot)}{2} \quad (5.2.2)$$

$$\pi_d = (p_d - \eta\frac{p_i + p_j}{2})q_d(\cdot) \quad (5.2.3)$$

with  $q_i(\cdot)$ ,  $q_j(\cdot)$ , and  $q_d(\cdot)$  as given by (4.4.8)–(4.4.10).

Equilibrium prices are:

$$\begin{aligned} p_u = \frac{1}{\omega} \{ & [(\beta - \gamma)(2(2\beta + 3\gamma) + \eta(\beta + \gamma))] \alpha \\ & + [(\beta + \gamma)\{2(2\beta + 3\gamma) - \eta(\beta + \gamma)\}] c_d \\ & + [2(\beta + \gamma)(2\beta + \gamma)] c_u \} \end{aligned} \quad (5.2.4)$$

$$\begin{aligned} p_d = \frac{1}{\omega} \{ & [(\beta - \gamma)(2(2\beta + 3\gamma) + \eta(2\beta + \gamma) + \eta^2(\beta + \gamma))] \alpha \\ & + [(\beta + \gamma)\{2(2\beta + 3\gamma) + \eta(2\beta - \gamma)\}] c_d \\ & + [(2\beta + \gamma)\{2\gamma + \eta(\beta + \gamma)\}] c_u \} \end{aligned} \quad (5.2.5)$$

where

$$\omega \equiv \eta^2(\beta + \gamma)^2 - 6\eta\gamma(\beta + \gamma) + 4\beta(2\beta + 3\gamma) \quad (5.2.6)$$

is used to simplify the equations. All coefficients on  $\alpha$ ,  $c_u$ , and  $c_d$  are positive for reasonable values of  $\eta$  under the initial assumption that  $\beta > \gamma > 0$ .<sup>43</sup>

The difference in downstream and upstream equilibrium prices is given by:

$$p_d - p_u = \frac{\eta(\beta - \gamma)(\beta + \eta(\beta + \gamma))\alpha - (2\beta + \gamma)(2\beta - \eta(\beta + \gamma))c_u + 3\eta\beta(\beta + \gamma)c_d}{\omega} \quad (5.2.7)$$

which is positive as long as  $\alpha$  is large relative to  $c_u$  and  $c_d$ , and  $\eta$  is not too close to 0.

Though quantities, profits, etc. can be derived, the equations quickly become cumbersome. As such, the following section concentrates on the special case  $\eta = 1$ .

<sup>43</sup> $\omega > 0$  is assured by the  $\beta > \gamma$  assumption:

$$\begin{aligned} \omega &= \beta^2(\eta^2 + 8) + \beta\gamma(2\eta^2 - 6\eta + 12) + \gamma^2(\eta^2 - 6\eta) \\ &> \gamma^2(2\eta^2 - 6\eta + 8) + \beta\gamma(2\eta^2 - 6\eta + 12) \\ &> 0 \end{aligned}$$

where the second step follows from  $\beta > \gamma$ , the third from  $\beta > \gamma > 0$ , and the fourth from the fact that the two polynomials in  $\eta$  attains strictly positive minimum values (at  $\eta = \frac{3}{2}$ ), and so are positive for all  $\eta$ .

The only possible negative coefficient in the price equations is on the  $c_d$  term in (5.2.4), but it can only be negative can if  $\eta > 4 + 2\frac{\gamma}{\beta + \gamma}$ , i.e. when the regulated cost is more than 4–5 times the upstream retail price.

### 5.2.1 Full retail price: $\eta = 1$

For this section, it is assumed that downstream firms must pay the full upstream retail price for upstream access, as in Telecom Decision CRTC 2010-802; in algebraic terms,  $\eta = 1$ .

Substituting this into (5.2.4)–(5.2.6) yields  $\omega_1 \equiv 9\beta^2 + 8\beta\gamma - 5\gamma^2$ , and corresponding prices of

$$p_u = \frac{1}{\omega_1} \left\{ [(\beta - \gamma)(5\beta + 7\gamma)]\alpha + [(\beta + \gamma)(3\beta + 5\gamma)]c_d + [2(\beta + \gamma)(2\beta + \gamma)]c_u \right\} \quad (5.2.8)$$

$$p_d = \frac{1}{\omega_1} \left\{ [(\beta - \gamma)(7\beta + 8\gamma)]\alpha + [(\beta + \gamma)(6\beta + 5\gamma)]c_d + [(2\beta + \gamma)(\beta + 3\gamma)]c_u \right\} \quad (5.2.9)$$

Comparing the upstream firms' price with the duopoly price from section 5.1, equation (5.1.4), yields:

$$p_u^{\eta=1} - p_u^{\text{duo}} = \frac{1}{\omega_1(2\beta - \gamma)} \left\{ [(\beta - \gamma)^2(\beta + 2\gamma)](\alpha - c_u) - [(3\beta^3 - 5\beta^2\gamma + 3\beta\gamma^2 + 5\gamma^3)]c_d \right\} \quad (5.2.10)$$

Using the initial assumption  $\alpha > c_u + c_d$ , the above difference is positive for any  $\gamma > \frac{1}{3}\beta$  (for larger values of  $\alpha$ ,  $\gamma$  can be closer to 0 while still resulting in the above difference being positive). Moreover,  $p_u^{\eta=1}$  *always* exceeds  $p_u^{\text{duo}}$  when  $\alpha > c_u + 3c_d$ , even when demand for the goods is entirely independent. Thus, unless the goods are very close to being independent and the maximum willingness-to-pay of consumers only slightly exceeds the marginal cost of the service (*each* of which seems an unreasonable assumptions in the context of internet service providers) prices of the upstream firms will be *higher* under the regulated scheme.

This result is not entirely surprising. The regulator, by tying downstream costs to upstream prices, gives upstream firms the power to increase the costs of one of their competitors, thus increasing their own demand by their own price. The usual decrease in quantity demanded effected by a price *increase* of an upstream firm is mitigated by an offsetting increase in quantity demanded caused by the increase in the price of one of the upstream firm's substitutes, namely the downstream firm's good.

As the goods become closer substitutes, i.e. as  $\gamma \rightarrow \beta$ :

$$p_u \rightarrow c_u + \frac{4}{3}c_d \quad (5.2.11)$$

$$p_d \rightarrow c_u + \frac{11}{6}c_d = p_u + \frac{1}{2}c_d \quad (5.2.12)$$

$$p_d^{\text{choke}} \rightarrow p_u \quad (5.2.13)$$

but this is clearly not possible, since  $p_d > p_d^{\text{choke}}$  and thus  $q_d < 0$ —the downstream firm does not participate in the market, and so the result is duopoly, with  $p_u \rightarrow c_u + c_d$ . Thus, when competition is strong because of close substitutability of the goods, setting the regulated price at the upstream retail price effectively mandates duopoly.

Assuming, however, that the goods are sufficiently differentiated that the downstream firm still participates, the resulting firm quantities are:

$$q_u^{\eta=1} = \frac{1}{\omega_1(\beta + 2\gamma)} \left\{ (2\beta + 3\gamma)(2\beta + \gamma)(\alpha - c_u) - \frac{(\beta + \gamma)(3\beta^2 - \beta\gamma - 5\gamma^2)}{\beta - \gamma} c_d \right\} \quad (5.2.14)$$

$$q_d^{\eta=1} = \frac{\beta + \gamma}{\omega_1(\beta + 2\gamma)} \left\{ (2\beta + \gamma)(\alpha - c_u) - \frac{6\beta^2 + 5\beta\gamma - 5\gamma^2}{\beta - \gamma} c_d \right\} \quad (5.2.15)$$

From which it follows<sup>44</sup> that  $q_u > 2q_d$ : the downstream firm will always supply less than half of what either upstream firm supplies; in total, less than  $\frac{1}{5}$  of the market.

Furthermore, the upstream firm (retail) quantities will always be lower than under duopoly. Since the downstream firm's price must now be below its choke price, the demand facing each upstream firm must be reduced, as can be readily seen from the upstream demand equations (4.4.8) and (4.4.9).

(5.2.15) also yields the downstream firm's participation constraint, which will be assumed to be satisfied and non-binding for the remainder of this section:

$$\begin{aligned} & q_d^{\eta=1} > 0 \\ \iff & c_d < \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} (\alpha - c_u) \end{aligned} \quad (5.2.16)$$

## 5.2.2 Comparison to duopoly

The change in profits of an upstream firm from duopoly to the full price regulation is:

$$\Delta\pi_u = \frac{1}{\xi_1} (a_1(\alpha - c_u)^2 + a_2(\alpha - c_u)c_d + a_3c_d^2) \quad (5.2.17)$$

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<sup>44</sup>The only difference in the coefficients on the  $(\alpha - c_u)$  terms of 5.2.14 and 5.2.15 is a factor of  $(2\beta + 3\gamma)$  vs.  $(\beta + \gamma)$ , which is clearly more than double. The coefficient on the  $c_d$  term of 5.2.15 is always negative, and always less than (i.e. more negative) than the second term of (5.2.14) (which can be either positive or negative); thus the total value of  $q_d$  is always less than half that of  $q_u$ .



where

$$\begin{aligned}\xi_1 &\equiv \omega_1^2(\beta + 2\gamma)(2\beta - \gamma)^2(\beta + \gamma) \\ a_1 &\equiv \frac{1}{2}(\beta - \gamma)^2(38\beta^5 + 86\beta^4\gamma + 84\beta^3\gamma^2 + 99\beta^2\gamma^3 + 30\beta\gamma^4 - 49\gamma^5) \\ a_2 &\equiv -42\beta^7 - 10\beta^6\gamma + 137\beta^5\gamma^2 + 190\beta^4\gamma^3 + 165\beta^3\gamma^4 - 125\beta^2\gamma^5 - 184\beta\gamma^6 + 85\gamma^7 \\ a_3 &\equiv -\frac{90\beta^8 + 516\beta^7\gamma + 516\beta^6\gamma^2 - 329\beta^5\gamma^3 - 551\beta^4\gamma^4 - 294\beta^3\gamma^5 + 150\beta^2\gamma^6 + 315\beta\gamma^7 - 125\gamma^8}{2(\beta - \gamma)}\end{aligned}$$

For  $\frac{\gamma}{\beta} \in (0, 1)$ ,  $a_1$  is always positive;  $a_2$  is negative between 0 and (approximately) 0.4426,<sup>45</sup> and positive between that value and 1; and  $a_3$  is always negative.<sup>46</sup>

This gives two cases for the change in profits:  $a_2 < 0$  and  $a_2 \geq 0$ , or equivalently,  $\frac{\gamma}{\beta} < 0.4426$  and  $\frac{\gamma}{\beta} \geq 0.4426$ .

Combining these two cases with the downstream participation inequality, (5.2.16), yields the two inequalities:

$$\Delta\pi_u > (\alpha - c_u)^2 \left[ a_1 + a_2 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right) + a_3 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right)^2 \right] \quad (5.2.18)$$

$$\Delta\pi_u > c_d^2 \left[ a_1 \left( \frac{6\beta^2 + 5\beta\gamma - 5\gamma^2}{(2\beta + \gamma)(\beta - \gamma)} \right)^2 + a_2 \left( \frac{6\beta^2 + 5\beta\gamma - 5\gamma^2}{(2\beta + \gamma)(\beta - \gamma)} \right) + a_3 \right] \quad (5.2.19)$$

both of which are strictly positive for  $\frac{\gamma}{\beta} \in (0, 1)$ .<sup>47</sup>

Thus the policy change from duopoly to full price-based regulation always results in strictly higher profits for the incumbent firms. Since total industry profits consist of the incumbent profits plus a positive profit for the downstream firm (since, by assumption, the downstream firm participates), total industry profits must also be higher.

The change in consumer surplus from duopoly to price-based regulation is:

$$\Delta CS = \frac{1}{\xi_1} b_1 (\alpha - c_u)^2 + b_2 (\alpha - c_u) c_d + b_3 c_d^2 \quad (5.2.20)$$

<sup>45</sup>See footnote 38 and preceding discussion for an alternative interpretation of  $\frac{\gamma}{\beta}$  values

<sup>46</sup> $a_1$  goes to positive infinity as  $\frac{\gamma}{\beta} \rightarrow -0.5^+$  and  $\frac{\gamma}{\beta} \rightarrow 2^-$ , and is strictly positive for all  $\frac{\gamma}{\beta}$  in between.

$a_2$  goes to negative infinity as  $\frac{\gamma}{\beta} \rightarrow -0.5^+$ , and has roots at 0.442626803845219 and 1.63223490537220.  $a_2$  is negative between  $-0.5$  and 0.442626803845219, and positive between the two roots.

$a_3$  has a root at  $-0.235196609205482$ , goes to infinity as  $\frac{\gamma}{\beta} \rightarrow 1^-$ , and is positive between the two values. Though this would appear to allow the change in profits go to infinity, this is ruled out by the downstream firm's participation constraint:  $\gamma$  cannot actually approach  $\beta$  while having the downstream firm still participate in the market; see the discussion after (5.2.13).

N.B. All three coefficients have other roots and/or infinite values, but all occur outside the roots/limits mentioned above. In this and following numerical analyses, roots and/or asymptotes that are irrelevant to the admissible  $(0, 1)$  values of  $\frac{\gamma}{\beta}$  are not reported.

<sup>47</sup>Both expressions are simply positively scaled versions of each other, with roots at  $\frac{\gamma}{\beta} = -0.2$  and 1, with no roots in between, and are positive between those two roots.

where

$$b_1 \equiv -\frac{(\beta - \gamma)(18\beta^6 + 102\beta^5\gamma + 74\beta^4\gamma^2 - 166\beta^3\gamma^3 - 133\beta^2\gamma^4 + 54\beta\gamma^5 + 15\gamma^6)}{2\xi_1} \quad (5.2.21)$$

$$b_2 \equiv \frac{18\beta^7 + 4\beta^6\gamma - 308\beta^5\gamma^2 - 460\beta^4\gamma^3 + 143\beta^3\gamma^4 + 327\beta^2\gamma^5 - 49\beta\gamma^6 - 35\gamma^7}{\xi_1} \quad (5.2.22)$$

$$b_3 \equiv \frac{54\beta^8 + 318\beta^7\gamma + 634\beta^6\gamma^2 - 58\beta^5\gamma^3 - 1113\beta^4\gamma^4 - 250\beta^3\gamma^5 + 604\beta^2\gamma^6 + 30\beta\gamma^7 - 75\gamma^8}{2\xi_1(\beta - \gamma)} \quad (5.2.23)$$

$b_1 > 0$  for  $\frac{\gamma}{\beta} \in (0, 0.9040]$ ,  $b_1 > 0$  for  $\frac{\gamma}{\beta} \in (0.9040, 1)$ ;  $b_2 > 0$  for  $\frac{\gamma}{\beta} \in (0, 0.2177]$  and  $b_2 < 0$  for  $\frac{\gamma}{\beta} \in (0.2177, 1)$ ;  $b_3 > 0$  for  $\frac{\gamma}{\beta} \in (0, 1)$ .<sup>48</sup> Thus there are 3 cases to consider: (i)  $\frac{\gamma}{\beta} \in (0, 0.2177)$ , (ii)  $\frac{\gamma}{\beta} \in (0.2177, 0.9040)$ , and (iii)  $\frac{\gamma}{\beta} \in (0.9040, 1)$ . Combined with the downstream firm participation constraint, (5.2.16), the first two cases yield a negative change in consumer surplus.<sup>49</sup> Case (iii) yields a negative change if  $c_d > \frac{\alpha - c_u}{165.29}$ —in other words, unless  $c_d$  is very small and the two goods are almost perfectly substitutable, consumer surplus will always be lower under the regulation than under duopoly.<sup>50</sup>

<sup>48</sup>The critical values given for  $b_1$  and  $b_2$  are, of course, approximate.

$b_1$  has roots at  $-0.231432490324345$ ,  $0.904037137771868$ , and  $1$ ; it is negative between the first two and positive between the last two roots.

$b_2$  has roots at  $-0.330664351201852$ ,  $0.217714363527372$ , and  $1.63212516510302$ ; it is positive between the first two and negative between the last two roots.

$b_3$  goes to positive infinity as  $\frac{\gamma}{\beta} \rightarrow -0.5^+$  and  $\frac{\gamma}{\beta} \rightarrow 1^-$ , and is positive in between.

All other roots/infinite values of the three coefficients lie outside the roots/limits mentioned above.

<sup>49</sup>For case (i):

$$\Delta CS < (\alpha - c_u)^2 \left\{ b_1 + b_2 \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} + b_3 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right)^2 \right\}$$

the right-hand side of which is negative in the range of  $\frac{\gamma}{\beta}$  under consideration.

For case (ii):

$$\Delta CS < c_d^2 \left\{ b_1 \left( \frac{6\beta^2 + 5\beta\gamma - 5\gamma^2}{(2\beta + \gamma)(\beta - \gamma)} \right)^2 + b_2 \left( \frac{6\beta^2 + 5\beta\gamma - 5\gamma^2}{(2\beta + \gamma)(\beta - \gamma)} \right) + b_3 \right\}$$

which, again, is always negative in the relevant  $\frac{\gamma}{\beta}$  range.

<sup>50</sup>Case (iii) yields:

$$\Delta CS < b_1(\alpha - c_u)^2 + \left( b_2 \frac{6\beta^2 + 5\beta\gamma - 5\gamma^2}{(2\beta + \gamma)(\beta - \gamma)} + b_3 \right) c_d^2$$

$b_1$  is positive in this case, while the coefficient on  $c_d$  is negative; thus  $\Delta CS$  is negative when the first term is smaller (in magnitude) than the second term, or when

$$\left( \frac{\alpha - c_u}{c_d} \right)^2 < \frac{- \left( b_2 \frac{6\beta^2 + 5\beta\gamma - 5\gamma^2}{(2\beta + \gamma)(\beta - \gamma)} + b_3 \right)}{b_1}$$

The right-hand side of the inequality attains a minimum of (approximately) 27321 at  $\frac{\gamma}{\beta} = 0.93627$ , thus

The change in total surplus is given by the following:

$$\Delta TS = d_1(\alpha - c_u)^2 + d_2(\alpha - c_u)c_d + d_3c_d^2$$

where

$$d_1 \equiv \frac{(\beta - \gamma)(90\beta^6 + 58\beta^5\gamma - 62\beta^4\gamma^2 + 164\beta^3\gamma^3 - 19\beta^2\gamma^4 - 208\beta\gamma^5 + 85\gamma^6)}{2\xi_1}$$

$$d_2 \equiv -\frac{162\beta^7 + 240\beta^6\gamma + 50\beta^5\gamma^2 - 144\beta^4\gamma^3 - 523\beta^3\gamma^4 + 5\beta^2\gamma^5 + 427\beta\gamma^6 - 145\gamma^7}{\xi_1}$$

$$d_3 \equiv \frac{162\beta^8 + 54\beta^7\gamma - 414\beta^6\gamma^2 - 584\beta^5\gamma^3 - 169\beta^4\gamma^4 + 1098\beta^3\gamma^5 + 284\beta^2\gamma^6 - 800\beta\gamma^7 + 225\gamma^8}{2\xi_1(\beta - \gamma)}$$

$d_1 > 0$ ,  $d_2 < 0$ , and  $d_3 \gtrless 0$  as  $\frac{\gamma}{\beta} \gtrless 0.5416$  for  $\frac{\gamma}{\beta} \in (0, 1)$ . Whether the change in total surplus is negative or positive depends on the values of  $\alpha$ ,  $c_u$ , and  $c_d$ . When  $c_d$  is close to 0, the positive  $d_1$  term dominates and the change in total surplus is positive; on the other hand, when the downstream firm participation constraint is close to binding, the change in total surplus is negative.

The reason for this counterintuitive possibility of increasing firm profits *and* increasing total surplus is that, while profits are increasing, this increase is due in part to the ability of upstream firms to increase their demand by earning profits from consumers whose preferences are better aligned with the downstream firm. Upstream prices increase and quantities decrease, but simultaneously the downstream firm's quantity increases (from 0) and price "decreases" (in effect, since the duopoly is equivalent to the three firm market with the downstream firm's price fixed at its demand choke price). Depending on the specific parameter values of the model, this "opening up" of the downstream firm's portion of the market can, as shown above, result in an increase in consumer surplus. Even when consumer surplus decreases, however, it is also possible that consumer surplus decreases by less than the increase in joint profits of the three firms.

Taking these results into account, the regulation on its own appears not entirely bad: it has the potential to create additional surplus in the market when compared to duopoly. A question arises, however, as to why such regulation is required at all. Since upstream profits strictly increase, while downstream profits increase from 0 (i.e. the downstream firm does not participate in the duopoly market), it would be in the interest of the firms to negotiate such a deal independently of the regulator. The critical distinction is that, in the context of this model, the regulator is actually aiding the upstream firms to collude: because the downstream firm must buy at the retail price, there is no competition between the upstream firms in the "downstream access" market.<sup>51</sup> Since a price-fixing agreement between the two firms would, presumably, be illegal, but following the regulator's decision would be legal, convincing the regulator to introduce such regulation would amount, in effect, to legally-sanctioned collusion.

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leading to  $\alpha - c_u < 165.29c_d$  being a sufficient (though non-necessary) condition for a negative change in consumer surplus.

<sup>51</sup>This could be mitigated by having the regulator-defined price set as a maximum price, rather than a fixed price. For simplification of the model in this paper, however, a separate "downstream access" market between upstream firms and the downstream firm is not considered.

### 5.3 Cost-based regulation

As an alternative to the regulated access cost being based on the retail price, in this section, an alternative pricing system, modeled after the CRTC's UBB replacement, is explored where the regulator sets upstream access pricing according to the upstream cost, i.e.  $c_r = \mu c_u$ . No particular bounds are placed on  $\mu$  (that is,  $\mu$  could be greater than or less than 1, to induce downstream marginal costs being higher or lower than the upstream marginal costs) except that it  $c_r$  is not so high that the downstream firm exits the market at equilibrium.<sup>52</sup> In the calculations that follow,  $\mu = 1$  is treated as a special case: the downstream firm's costs are exactly equal to the upstream firm's costs, and the upstream firms are exactly compensated for the cost incurred by downstream customers. As will be shown,  $\mu = 1$  is better than any higher value of  $\mu$  (although, as will be shown, the optimal  $\mu$  is actually strictly less than 1). Since under  $\mu = 1$  all three firms are identical in every way, this special case is simply an identical-firm, differentiated-product Bertrand equilibrium.

Since  $\mu c_u$  is now exogenous, the only influence an upstream firm has on a downstream firm is through the effect of its price on the downstream firm's demand, unlike the previous section where an upstream firm's price affected both the downstream firm's demand *and* the downstream firm's marginal cost.<sup>53</sup>

Profits of the firms are:

$$\pi_i^{\text{cost}} = (p_i - c_u - c_d)q_i^{\text{cost}}(\cdot) + (\mu - 1)c_u q_d^{\text{cost}}(\cdot) \quad (5.3.1)$$

$$\pi_d^{\text{cost}} = (p_d - \mu c_u - c_d)q_d^{\text{cost}}(\cdot) \quad (5.3.2)$$

(with  $p_j^{\text{cost}}$  symmetric to  $p_i^{\text{cost}}$ ).

Equilibrium prices are given by:

$$p_u = \frac{\beta - \gamma}{2\beta}\alpha + \frac{\beta + \gamma}{2\beta}c_d + \frac{(\beta + \gamma)(2\beta + \gamma + 2\gamma\mu)}{2\beta(2\beta + 3\gamma)}c_u \quad (5.3.3)$$

$$p_d = \frac{\beta - \gamma}{2\beta}\alpha + \frac{\beta + \gamma}{2\beta}c_d + \frac{\gamma(2\beta + \gamma) + (2\beta^2 + 3\beta\gamma + 2\gamma^2)\mu}{2\beta(2\beta + 3\gamma)}c_u \quad (5.3.4)$$

(where  $p_u \equiv p_i = p_j$ ). The latter term can also be written as:

$$p_d = p_u + (\mu - 1) \left( \frac{2\beta + \gamma}{2(2\beta + 3\gamma)} \right) c_u \quad (5.3.5)$$

from which it is apparent that the downstream firm's prices are greater than, equal to, or less than the upstream firms' prices as  $\mu$  is greater than, equal to, or less than 1.

<sup>52</sup>In algebraic terms, since there are no fixed costs in this model, the downstream firm enters so long as  $\mu c_u + c_d < p_d^{\text{choke}}$ , where  $p_d^{\text{choke}}$  is given by (5.1.1). A prohibitively high regulated cost violating this condition would, in effect, amount to the regulator choosing the duopoly option explored in the section 5.1.

<sup>53</sup>Also see section 7.4 for an idea for an interesting extension to this model which would allow a firm, under this regulation structure, to make an investment decision that alters its marginal costs, allowing the downstream firm to be affected by the upstream firm's cost-reducing (or cost-increasing) investment decision.

Solving for equilibrium quantities yields:

$$q_u = \frac{\beta + \gamma}{2\beta(\beta + 2\gamma)} (\alpha - c_d) - \frac{(2\beta + \gamma)(\beta^2 + \beta\gamma - \gamma^2) - \mu\gamma^2(\beta + 2\gamma)}{2\beta(\beta - \gamma)(\beta + 2\gamma)(2\beta + 3\gamma)} c_u \quad (5.3.6)$$

$$q_d = \frac{\beta + \gamma}{2\beta(\beta + 2\gamma)} (\alpha - c_d) + \frac{(\beta + \gamma)(\gamma(2\beta + \gamma) - \mu(\beta + 2\gamma)(2\beta - \gamma))}{2\beta(\beta - \gamma)(\beta + 2\gamma)(2\beta + 3\gamma)} c_u \quad (5.3.7)$$

(where  $q_u \equiv q_i = q_j$ ). Since there are no fixed costs in the model to consider, these equations yield the participation constraints for upstream and downstream firms:

$$\mu > \mu_u^{\text{crit}} = \frac{(2\beta + \gamma)(\beta^2 + \beta\gamma - \gamma^2) - (\beta - \gamma)(\beta + \gamma)(2\beta + 3\gamma) \left(\frac{\alpha - c_d}{c_u}\right)}{\gamma^2(\beta + 2\gamma)} \quad (5.3.8)$$

$$\mu < \mu_d^{\text{crit}} = \frac{\gamma(2\beta + \gamma) + (\beta - \gamma)(2\beta + 3\gamma) \left(\frac{\alpha - c_d}{c_u}\right)}{(\beta + 2\gamma)(2\beta - \gamma)} \quad (5.3.9)$$

Using assumptions (4.4.2) and (4.4.3),  $\mu_u^{\text{crit}} < 1$  and  $\mu_d^{\text{crit}} > 1$ , so there always exists a range of values of  $\mu$  which includes  $\mu = 1$  that induce participation in the market by both upstream and downstream firms. It is also worth noting that  $\mu_u^{\text{crit}}$  is not necessarily positive: in particular, when  $\frac{\alpha - c_d}{c_u}$  is large, the upstream firms would still participate even if they had to *pay* some fraction of the cost (in addition to incurring the cost) to the downstream firm for each unit the downstream firm sells.

The equilibrium quantity  $q_d$  can also be expressed relatively to the upstream quantity  $q_u$ :

$$q_d = q_u - (\mu - 1) \left( \frac{2\beta + \gamma}{2(\beta - \gamma)(2\beta + 3\gamma)} \right) c_u \quad (5.3.10)$$

from which, combined with (5.3.5), it is clear that, at equilibrium,  $p_d \gtrless p_i = p_j$  and  $q_d \lesseqgtr q_i = q_j$  as  $\mu \gtrless 1$ . This, of course, is entirely intuitive: at  $\mu = 1$  all three firms are identical in every way.

Firm profits at equilibrium are, from (5.3.1) and (5.3.2):

$$\begin{aligned}
\pi_u &= \frac{(\beta + \gamma)(\beta - \gamma)}{4\beta^2(\beta + 2\gamma)}(\alpha - c_d)^2 \\
&+ \frac{(\alpha - c_d)c_u}{4\beta^2(2\beta + 3\gamma)} \left\{ \mu(2\beta^2 + 3\beta\gamma + 2\gamma^2) - \frac{6\beta^3 + 13\beta^2\gamma + 4\beta\gamma^2 - 2\gamma^3}{\beta + 2\gamma} \right\} \\
&+ \frac{c_u^2}{4\beta^2(\beta - \gamma)(2\beta + 3\gamma)^2} \left\{ \left[ \frac{(2\beta + \gamma)(2\beta^4 + 8\beta^3\gamma + \beta^2\gamma^2 - 3\beta\gamma^3 + 2\gamma^4)}{\beta + 2\gamma} \right] \right. \\
&\quad \left. + \mu[4\beta^4 + 8\beta^3\gamma + \beta^2\gamma^2 - 3\beta\gamma^3 + 2\gamma^4] \right. \\
&\quad \left. - \mu^2[(2\beta + \gamma)(\beta + \gamma)(2\beta^2 + \beta\gamma - 2\gamma^2)] \right\} \quad (5.3.11)
\end{aligned}$$

$$\begin{aligned}
\pi_d &= \frac{(\beta + \gamma)(\beta - \gamma)}{4\beta^2(\beta + 2\gamma)}(\alpha - c_d)^2 + \frac{c_u(\alpha - c_d)(\beta + \gamma)}{2\beta^2(2\beta + 3\gamma)} \left\{ \frac{\gamma(2\beta + \gamma)}{\beta + 2\gamma} - \mu(2\beta - \gamma) \right\} \\
&+ \frac{c_u^2(\beta + \gamma)}{4\beta^2(\beta - \gamma)(2\beta + 3\gamma)^2} \left\{ \left[ \frac{\gamma^2(2\beta + \gamma)^2}{\beta + 2\gamma} \right] \right. \\
&\quad \left. - \mu[2\gamma(2\beta + \gamma)(2\beta - \gamma)] + \mu^2[(\beta + 2\gamma)(2\beta - \gamma)^2] \right\} \quad (5.3.12)
\end{aligned}$$

For the special case  $\mu = 1$ , profits are equal:

$$\pi_u^{\mu=1} = \pi_d^{\mu=1} = \frac{(\beta + \gamma)(\beta - \gamma)}{4\beta^2(\beta + 2\gamma)}(\alpha - c_d - c_u)^2 \quad (5.3.13)$$

Consumer surplus at equilibrium is:

$$\begin{aligned}
CS &= \frac{3(\beta + \gamma)^2}{8\beta^2(\beta + 2\gamma)}(\alpha - c_d)^2 - \frac{c_u(\alpha - c_d)(\beta + \gamma)}{4\beta^2} \left\{ \frac{2\beta + \gamma}{\beta + 2\gamma} + \mu \right\} \\
&+ \frac{(\beta + \gamma)c_u^2}{8\beta^2(\beta - \gamma)(2\beta + 3\gamma)^2} \left\{ \left[ \frac{(2\beta + \gamma)^2(2\beta^2 + 2\beta\gamma - 3\gamma^2)}{\beta + 2\gamma} \right] \right. \\
&\quad \left. + \mu[2\gamma(2\beta - 3\gamma)(2\beta + \gamma)] + \mu^2[4\beta^3 + 3\beta^2\gamma + \beta\gamma^2 - 6\gamma^3] \right\} \quad (5.3.14)
\end{aligned}$$

At equal costs,  $\mu = 1$ , this simplifies to:

$$CS^{\mu=1} = \frac{3(\beta + \gamma)^2}{8\beta^2(\beta + 2\gamma)}(\alpha - c_d - c_u)^2 \quad (5.3.15)$$

Putting together firm profits and consumer surplus yields total surplus:

$$\begin{aligned}
TS &= CS + \pi_i + \pi_j + \pi_d \\
&= \frac{3(3\beta - \gamma)(\beta + \gamma)}{8\beta^2(\beta + 2\gamma)}(\alpha - c_d)^2 - \frac{c_u(\alpha - c_d)}{4\beta^2} \left\{ \frac{8\beta^2 + 5\beta\gamma - \gamma^2}{\beta + 2\gamma} + \mu(\beta - \gamma) \right\} \\
&\quad + \frac{c_u^2}{8\beta^2(\beta - \gamma)(2\beta + 3\gamma)^2} \left\{ \left[ \frac{(2\beta + \gamma)(12\beta^4 + 22\beta^3\gamma - 14\beta^2\gamma^2 - 29\beta\gamma^3 + 3\gamma^4)}{\beta + 2\gamma} \right] \right. \\
&\quad \quad \quad \left. + \mu[2(8\beta^4 + 12\beta^3\gamma - 6\beta^2\gamma^2 - 11\beta\gamma^3 + 3\gamma^4)] \right. \\
&\quad \quad \quad \left. - \mu^2[(\beta + \gamma)(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)] \right\} \quad (5.3.16)
\end{aligned}$$

With  $\mu = 1$ , total surplus simplifies to:

$$TS^{\text{cost}, \mu=1} = \frac{3(\beta + \gamma)(3\beta - \gamma)}{8\beta^2(\beta + 2\gamma)}(\alpha - c_d - c_u) \quad (5.3.17)$$

The socially optimal  $\mu$  is the solution to:

$$\max_{\mu \in [\mu_u^{\text{crit}}, \mu_d^{\text{crit}}]} TS \quad (5.3.18)$$

which is:<sup>54</sup>

$$\mu^* = \frac{8\beta^4 + 12\beta^3\gamma - 6\beta^2\gamma^2 - 11\beta\gamma^3 + 3\gamma^4}{(\beta + \gamma)(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)} - \frac{(\beta - \gamma)^2(2\beta + 3\gamma)^2}{(\beta + \gamma)(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)} \left( \frac{\alpha - c_d}{c_u} \right) \quad (5.3.19)$$

which, under model assumptions (4.4.2) and (4.4.3), satisfies upstream and downstream participation constraints:  $\mu_u^{\text{crit}} < \mu^* < \mu_d^{\text{crit}}$ .

Rearranging the initial assumption  $\alpha > c_d + c_u$  into  $\frac{\alpha - c_d}{c_u} > 1$ , and noting that all of the polynomials of  $\beta$  and  $\gamma$  in (5.3.19) are positive for  $\beta > \gamma > 0$ , it follows that:

$$\begin{aligned}
\mu^* &< \frac{8\beta^4 + 12\beta^3\gamma - 6\beta^2\gamma^2 - 11\beta\gamma^3 + 3\gamma^4}{(\beta + \gamma)(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)} - \frac{(\beta - \gamma)^2(2\beta + 3\gamma)^2}{(\beta + \gamma)(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)} \quad (1) \\
\mu^* &< \frac{(\beta + \gamma)(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)}{(\beta + \gamma)(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)} \\
\mu^* &< 1
\end{aligned}$$

<sup>54</sup>The second order condition for a maximum is easily satisfied here:

$$\frac{\partial^2 TS}{\partial \mu^2} = \frac{-2[(\beta + \gamma)(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)]c_u^2}{8\beta^2(\beta - \gamma)(2\beta + 3\gamma)^2}$$

which is negative because  $c_u > 0$  and  $\beta > 0$  by initial assumption, and each of  $(\beta - \gamma)$ ,  $(2\beta + 3\gamma)$ ,  $(\beta + \gamma)$ , and  $(4\beta^3 + 4\beta^2\gamma + \beta\gamma^2 - 6\gamma^3)$  is guaranteed positive by the assumption  $\beta > \gamma > 0$ .

i.e. that the regulated upstream access cost for the downstream firm should always be *less than* the marginal cost of that access incurred by the upstream firm. It is also possible for the optimal  $\mu$  to be negative—particularly if the  $\alpha - c_d$  is large relative to  $c_u$ , as then the negative last term of (5.3.19) dominates to make  $\mu^*$  negative while the positive first term of (5.3.11) dominates to keep  $\pi_u$  positive.

In effect, by subsidizing the costs of the downstream firm at the expense of the upstream firms, the regulator is able to lower the equilibrium prices of all three firms, thus increasing consumer surplus by more than the associated loss in profits. This result is not entirely unexpected: since all firms have some market power in this model, prices will always be above marginal cost. Because the best response of each of the three firms to a price decrease of another firm is to decrease the firm's own price, any regulation that effects a price reduction in one firm (in this case by reducing costs of the downstream firm, which lowers the downstream firm's optimal price) reduces the price of all three firms, thus reducing deadweight loss compared to the equal cost case  $\mu = 1$ .

It is also worth noting that this result depends significantly on the presence of only a single downstream firm. As additional downstream firms enter the market, the upstream firms' market power would be eroded, and the optimal  $\mu$  would necessarily increase to induce non-negative profits for the incumbents, approaching a limit of  $\mu = 1$  as the market approaches perfect competition.

### 5.3.1 Comparison to duopoly

A regulator may be unwilling—because, for example, it anticipates significant additional downstream firm entry, making the market strongly competitive—or unable—perhaps for political reasons—to set  $\mu < 1$ . In the analysis that follows, it will be assumed that this is the case, and thus a regulator will set the best value of  $\mu$  available to it:  $\mu = 1$ . For a regulator sufficiently bold to set downstream costs below actual upstream costs, consumer surplus, downstream profits and total surplus would be higher while upstream profits would be lower.

Compared to duopoly, the cost-based regulation results in lower upstream prices, quantities, and profits under the model assumptions:

$$\Delta p_u = -\frac{\gamma(\beta - \gamma)}{2\beta(2\beta - \gamma)}(\alpha - c_u - c_d) \quad (5.3.20)$$

$$\Delta q_u = -\frac{\gamma(\beta^2 + \gamma^2)}{2\beta(\beta + \gamma)(2\beta - \gamma)(\beta + 2\gamma)}(\alpha - c_u - c_d) \quad (5.3.21)$$

$$\Delta \pi_u = -\frac{\gamma(\beta - \gamma)(4\beta^3 + 3\beta^2\gamma + 2\beta\gamma^2 - \gamma^3)}{4\beta^2(\beta + 2\gamma)(\beta + \gamma)(2\beta - \gamma)^2}(\alpha - c_u - c_d)^2 \quad (5.3.22)$$

Consumer surplus, meanwhile, increases from duopoly:

$$\Delta CS = \frac{(\beta - \gamma)(4\beta^4 + 12\beta^3\gamma + 15\beta^2\gamma^2 - 3\gamma^4)}{(8\beta^2(\beta + 2\gamma)(\beta + \gamma)(2\beta - \gamma)^2)}(\alpha - c_u - c_d)^2 \quad (5.3.23)$$



When  $\frac{\gamma}{\beta} \leq 0.7160$ , this increase to consumer surplus exceeds the loss of upstream profits; when total surplus is considered, by also adding downstream profits, the result is always a strictly positive change to total surplus:

$$\Delta TS = \Delta CS + 2\Delta\pi_u + \pi_d \quad (5.3.24)$$

$$= \frac{(\beta - \gamma)(12\beta^4 + 4\beta^3\gamma - 3\beta^2\gamma^2 - 12\beta\gamma^3 + 3\gamma^4)}{\beta^2(\beta + 2\gamma)(\beta + \gamma)(2\beta - \gamma)^2} (\alpha - c_u - c_d)^2 \quad (5.3.25)$$

### 5.3.2 Comparison to full price regulation

Compared to the full retail price regulation, profits of the upstream firms are lower (they were already higher than duopoly under the retail price regulation—see discussion following (5.2.17)). Downstream price decreases and quantity increases:

$$p_d^{\text{cost}} - p_d^{\text{price}} = -\frac{(\beta - \gamma)(5\beta^2 + 8\beta\gamma + 5\gamma^2)(\alpha - c_u) + (\beta + \gamma)(3\beta^2 + 2\beta\gamma + 5\gamma^2)c_d}{2\beta(9\beta^2 + 8\beta\gamma - 5\gamma^2)} \quad (5.3.26)$$

$$q_d^{\text{cost}} - q_d^{\text{price}} = \frac{(\beta + \gamma)(5\beta^2 + 6\beta\gamma - 5\gamma^2)(\alpha - c_u)}{2\beta(\beta + 2\gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)} + \frac{(\beta + \gamma)^2(3\beta^2 + 8\beta\gamma - 5\gamma^2)c_d}{2\beta(\beta - \gamma)(\beta + 2\gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)} \quad (5.3.27)$$

Profits of the downstream firm increase by:

$$\pi_d^{\text{cost}} - \pi_d^{\text{price}} = A_1(\alpha - c_u)^2 + A_2(\alpha - c_u)c_d + A_3 \quad (5.3.28)$$

where

$$A_1 \equiv \frac{(\beta + \gamma)(\beta - \gamma)(13\beta^2 + 10\beta\gamma - 5\gamma^2)(5\beta^2 + 6\beta\gamma - 5\gamma^2)}{4(\beta + 2\gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2\beta^2}$$

$$A_2 \equiv -\frac{(\beta + \gamma)(33\beta^5 - \beta^4\gamma - 150\beta^3\gamma^2 - 34\beta^2\gamma^3 + 105\beta\gamma^4 - 25\gamma^5)}{2(\beta + 2\gamma)\beta^2(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2}$$

$$A_3 \equiv -\frac{(\beta + \gamma)^2(3\beta^2 + 8\beta\gamma - 5\gamma^2)(21\beta^3 + 9\beta^2\gamma - 23\beta\gamma^2 + 5\gamma^3)}{4(\beta + 2\gamma)(\beta - \gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2\beta^2}$$

Assuming the downstream firm's participation constraint was satisfied under the price-based regulation, and incorporating the initial assumptions (4.4.2) and (4.4.3), the change in downstream profits, (5.3.28), is always positive.<sup>55</sup>

<sup>55</sup>Proof:  $A_1 > 0$  and  $A_3 < 0$  for all  $\frac{\gamma}{\beta} \in (0, 1)$ .  $A_2$  is negative when  $\frac{\gamma}{\beta} < 0.4733$  (approximately), and positive when  $\frac{\gamma}{\beta} \geq 0.4733$ . The former case yields

$$\Delta\pi_d > \left[ A_1 + A_2 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right) + A_3 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right)^2 \right] (\alpha - c_u)^2$$

while the second case yields the inequality with the right-hand-side multiplied by the positive value  $\left( \frac{6\beta^2 + 5\beta\gamma - 5\gamma^2}{(2\beta + \gamma)(\beta - \gamma)} \right)^2 \left( \frac{c_d}{\alpha - c_u} \right)^2$ . Thus both have the same sign for any given  $\frac{\gamma}{\beta}$ : in this case, the expression goes to positive infinity at  $\frac{\gamma}{\beta} = -0.5$  and  $\frac{\gamma}{\beta} = 1$ , with no roots in between, and so the whole expression is positive for  $\frac{\gamma}{\beta} \in (0, 1)$ .

The change in total surplus is given by:

$$TS^{\text{cost}} - TS^{\text{price}} = B_1(\alpha - c_u)^2 + B_2(\alpha - c_u)c_d + B_3c_d^2 \quad (5.3.29)$$

where

$$\begin{aligned} B_1 &\equiv \frac{(\beta - \gamma)(153\beta^5 + 455\beta^4\gamma + 182\beta^3\gamma^2 - 478\beta^2\gamma^3 - 315\beta\gamma^4 + 75\gamma^5)}{8\beta^2(\beta + 2\gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2} \\ B_2 &\equiv -\frac{81\beta^6 + 30\beta^5\gamma - 497\beta^4\gamma^2 - 476\beta^3\gamma^3 + 403\beta^2\gamma^4 + 390\beta\gamma^5 - 75\gamma^6}{4\beta^2(\beta + 2\gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2} \\ B_3 &\equiv \frac{3(\beta + \gamma)(27\beta^6 - 36\beta^5\gamma - 65\beta^4\gamma^2 + 208\beta^3\gamma^3 + 69\beta^2\gamma^4 - 180\beta\gamma^5 + 25\gamma^6)}{8\beta^2(\beta + 2\gamma)(\beta - \gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2} \end{aligned}$$

This change is always positive.<sup>56</sup>

Consumer surplus changes by:

$$CS^{\text{cost}} - CS^{\text{price}} = C_1(\alpha - c_u)^2 + C_2(\alpha - c_u)c_d + C_3c_d^2 \quad (5.3.30)$$

where

$$\begin{aligned} C_1 &\equiv \frac{3(\beta + \gamma)(\beta - \gamma)(33\beta^4 + 130\beta^3\gamma + 156\beta^2\gamma^2 + 30\beta\gamma^3 - 25\gamma^4)}{8((\beta + 2\gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2\beta^2)} \\ C_2 &\equiv -\frac{(\beta + \gamma)(99\beta^5 + 211\beta^4\gamma - 122\beta^3\gamma^2 - 458\beta^2\gamma^3 - 165\beta\gamma^4 + 75\gamma^5)}{4((\beta + 2\gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2\beta^2)} \\ C_3 &\equiv \frac{3(\beta + \gamma)^2(9\beta^5 - 49\beta^4\gamma - 110\beta^3\gamma^2 + 46\beta^2\gamma^3 + 105\beta\gamma^4 - 25\gamma^5)}{8((\beta + 2\gamma)(9\beta^2 + 8\beta\gamma - 5\gamma^2)^2\beta^2(\beta - \gamma))} \end{aligned}$$

This change to consumer surplus is always positive.<sup>57</sup>

In summary, compared to full retail price UBB, equal-cost regulation increases consumer surplus, downstream firm profits, and total surplus, though reduces upstream firm profits.

<sup>56</sup>Proof:  $B_1 > 0$  and  $B_3 > 0$  for all  $\frac{\gamma}{\beta} \in (0, 1)$ .  $B_2 \geq 0$  as  $\frac{\gamma}{\beta} \geq 0.3933$  (approximately). If  $B_2$  is positive, then clearly (5.3.29) is positive since every term is positive. If  $B_2$  is negative, incorporating the downstream participation constraint (5.2.16) yields:

$$\Delta TS > \left[ B_1 + B_2 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right) \right] (\alpha - c_u)^2 + B_3c_d^2$$

The term inside square brackets has roots at  $\frac{\gamma}{\beta} = -0.3560$  (approx.) and 1, and is strictly positive between those roots. Thus all terms are positive, and so the total change in surplus is positive for all  $\frac{\gamma}{\beta} \in (0, 1)$ .

<sup>57</sup>Proof:  $C_1 > 0$  for all  $\frac{\gamma}{\beta} \in (0, 1)$ .  $C_2 \leq 0$  as  $\frac{\gamma}{\beta} \leq 0.7036$  (approx.).  $C_3 \geq 0$  as  $\frac{\gamma}{\beta} \leq 0.1420$  (approx.). This then yields 3 cases.

Case 1:  $\frac{\gamma}{\beta} \in (0, 0.1420)$ :  $C_1 > 0, C_2 < 0, C_3 > 0$ .

$$\Delta CS > \left[ C_1 + C_2 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right) \right] (\alpha - c_u)^2 + C_3c_d^2$$

The term inside square brackets is positive under the price-based downstream participation constraint (5.2.16), as is  $C_3c_d^2$ , and so the change to  $CS$  is positive.

## 5.4 Comparison of UBB and cost-based mechanisms

The UBB analysis of section 5.2.1 considered only the case where the regulated price is equal to the full upstream retail price, i.e.  $\eta = 1$ . One remaining question, however, is whether the difference between UBB and a cost-based approach is just a matter of the wrong choice of  $\eta$ , the retail price multiple. In other words, can UBB, with a sufficiently low  $\eta$ , achieve the same result as the cost-based approach with  $\mu = 1$ ?

The answer is that, in general, it cannot: in order to achieve the  $\mu = 1$  outcome, both of the following equations must be solved through a choice of  $\eta$ :

$$p_d^\eta = p_d^{\mu=1} \quad (5.4.1)$$

$$p_u^\eta = p_d^\eta \quad (5.4.2)$$

(Note that solving (5.4.2) is equivalent to solving for a value of  $\eta$  that makes equation (5.2.7) equal 0). There are, however, two reasons that this cannot be achieved.

First, from a purely mathematical approach, there are two distinct equations with only one unknown variable: no single value of  $\eta$  can, in general, solve both equations. Second, taking an upstream firm's profit function, (5.2.1), repeated below, and differentiating with respect to its price yields:

$$\pi_i = (p_i - c_u - c_d)q_i(\cdot) + (\eta p_i - c_u)\frac{q_d(\cdot)}{2} \quad (5.2.1)$$

$$\frac{\partial \pi_i}{\partial p_i} = q_i(\cdot) - p_i \left( \frac{\beta + \gamma}{(\beta + 2\gamma)(\beta - \gamma)} \right) + \eta \frac{q_d(\cdot)}{2} + \frac{(\eta p_i - c_u)}{2} \left( \frac{\gamma}{(\beta + 2\gamma)(\beta - \gamma)} \right) \quad (5.4.3)$$

To be the same as the  $\mu = 1$  equilibrium, this expression must be equal to 0 at equilibrium prices. However, from the  $\mu = 1$  equilibrium, the first two terms equal 0 at the supposed equilibrium values. Thus the second two terms, involving  $\eta$ , must also equal 0 for this to be an equilibrium price for the upstream firm. Since the third term is strictly positive (the downstream firm participates by assumption), this means the fourth term must be negative, which can only be the case if  $c_r \equiv \eta p_i < c_u$ . That, however, means the cost to the

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Case 2:  $\frac{\gamma}{\beta} \in (0.1420, 0.7036)$ :  $C_1 > 0, C_2 < 0, C_3 < 0$ .

$$\Delta CS > \left[ C_1 + C_2 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right) + C_3 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right)^2 \right] (\alpha - c_u)^2$$

Assuming the downstream firm participates under the price-based regulation, the term inside square brackets is positive for  $\frac{\gamma}{\beta} \in (0, 1)$ ; thus  $CS$  increases.

Case 3:  $\frac{\gamma}{\beta} \in (0.7036, 1)$ :  $C_1 > 0, C_2 > 0, C_3 < 0$ .

$$\Delta CS > C_2(\alpha - c_u)c_d + \left[ C_1 + C_3 \left( \frac{(2\beta + \gamma)(\beta - \gamma)}{6\beta^2 + 5\beta\gamma - 5\gamma^2} \right)^2 \right] (\alpha - c_u)^2$$

which, again under (5.2.16), is positive for  $\frac{\gamma}{\beta} \in (0, 1)$ .

Thus consumer surplus increases for all  $\frac{\gamma}{\beta} \in (0, 1)$ .

downstream firm is less than  $c_u$ , which is equivalent, from the downstream firm's point of view, to a cost-based approach with  $\mu < 1$ . Putting this into (5.3.5)<sup>58</sup> immediately reveals that the downstream firm will optimally set a price lower than the upstream firms.

Thus under UBB, the regulator can either lower the downstream firm's price to what it would be under the cost-based approach, but have the upstream prices higher than the cost-based approach, or it can lower the upstream firm's prices to the cost-based approach by setting the access cost below the actual cost, having the downstream firm's price below its level under the cost-based approach. The latter, however, was already ruled out after (5.3.20) by assuming the regulator unwilling to regulate below-cost access, which led to  $\mu = 1$  even though  $\mu < 1$  was socially optimal. The cost-based approach's equilibrium of having the firms compete in the market as equal competitors simply is not possible: without subsidizing the downstream firm, by tying the downstream firm's cost to the retail price of the upstream firms, the UBB approach always gives upstream firms an incentive to raise price over a comparable cost-based equilibrium.

## 6 Conclusion

UBB provides an interesting insight into the decision making process of the CRTC. Rather than operating from principles of outcomes, the CRTC regulations as concerning UBB depict a set of regulations that, while each are individually justifiable, combine to produce an inefficient outcome.

As the model presented in this paper has shown, UBB has the potential to be substantially worse than a cost-based pricing approach, resulting in higher profits, lower consumer surplus, and lower overall economic surplus. Moreover UBB, as presented in this model, has the potential to reduce the overall welfare of the economy as compared to leaving a duopoly unregulated.

In the end, the CRTC, on the brink of being overruled by the Canadian Parliament, retracted UBB and came up with a cost-based approach. Doing so was an important step in the mechanism of the of the ISP market in Canada, for it severed the link between incumbent pricing and independent ISP costs, thereby eliminating the extra incentive for incumbent ISPs to raise prices above their duopoly level. The cost-based approach enacted, however, was not without problems: as shown in this model, socially optimal pricing requires access costs to be set at least as low as marginal costs, and possibly lower; by not requiring stringent cost justifications and accepting a nearly ten-fold difference in pricing across ISPs—even across ISPs using the same basic technology with comparable geographic constraints—it seems likely that the costs enacted by the CRTC for most firms<sup>59</sup> are still much higher

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<sup>58</sup>Though (5.3.5) is for the cost-based approach, since we are comparing with the cost-based equilibrium, and the upstream prices and quantities are assumed equal to their values under the cost-based approach, this approach is numerically valid.

<sup>59</sup>MTS Allstream, which reported costs significantly lower than any of the other incumbents, is a possible exception.

than the actual costs incurred by incumbents. Although in the end the CRTC enacted a better alternative to UBB, substantial gains are still to be made by more accurately—and independently—determining appropriate costs.

UBB, as modeled in this paper, has little to recommend it. It is anticompetitive, heavily favours incumbents over independent ISPs, and, rather than lowering prices through competition, actually resulted in increased prices as compared to a simple duopoly. That the CRTC lacked the economic expertise to recognize the faults in linking retail prices to independent ISP costs calls into question the CRTC's ability to make decisions that are, ultimately, about industrial competition rather than regulation of telecommunications protocols. Since, by entering into cost regulation of the Canadian ISP market, the CRTC has become not only a telecommunications bookkeeper and conflict resolver but also a determiner of competitive structure in the telecommunications field, it would behoove the Canadian government to require the Commission to engage in consultation with independent economic advisors to properly understand the likely economic consequences of its future decisions.

It remains to be seen whether the cost-based replacement introduced by the CRTC and modeled in this paper is able to offer the sort of competition needed to reduce prices compared to other OECD countries. While a cost-based approach is almost certainly superior to UBB, even at significant price discounts (as discussed in section 5.4), it seems likely that the system can be improved by matching regulatory access pricing with the true costs of access. Moreover, according to the optimal cost multiplier in (5.3.20), it appears better—overall, and for everyone except incumbent firms—to err on the side of costs being too low rather than too high, at least until a substantial level of competition emerges. While the decision to replace UBB with a cost-based approach should be praised, it should not be overlooked that there still exists a potential opportunity for the CRTC to further increase the competitiveness of the ISP market by ensuring that regulated pricing is closely aligned with the true ISP costs of access and related investment. Evidence from differences in cost submissions, the lack of independent verification of those submissions, and the CRTC policy of a premium of 15% over approved costs strongly suggests that, while a preferable mechanism has been instituted, costs are still significantly too high, reducing the benefits of competition and limiting the available gains to economic surplus of internet access in Canada.

## 7 Future extensions

Several ideas arose during the creation of this paper. Some of these were incorporated into the model, while others were originally a part of the model but were removed for tractability. Still other ideas presented themselves as natural extensions of the model and relaxation of the model assumptions. Some of the more intriguing ideas left out of the model are noted briefly below.

## 7.1 Upstream investment decision

Early versions of this paper had an investment decision by each upstream firm instead of assuming such investment to be included in  $c_u$ . Investment had two components (assumed at a fixed ratio, for simplicity): an “advertising” component that did not make the upstream firm’s technology any better, but transferred demand from the other upstream competitor; and a “technology” component that improved the firm’s network, thus increasing the firm’s demand. The downstream firm was not affected by the advertising component, since customers transferring from one technology to the other were assumed to remain with the downstream firm but received the benefit of each firm’s technology investment. The marginal cost of investment was positive and increasing.

Though such an addition makes for a more interesting model when questions about firm investment decisions are sought, it adds little to the analysis of UBB versus the cost-based approach as analysed in this paper, but much to the complexity, and so was removed; any such investment was instead assumed to scale linearly with the number of users, and thus notionally included in  $c_u$ .

## 7.2 Multiple downstream firms

The model as structured in this paper has only a single downstream firm. It would be worth considering a modified version of the model where  $q_d$  is actually an aggregate of downstream firms operating as a competitive fringe without differentiation between downstream firms, while maintaining the differentiation of the incumbents versus each other and the fringe as a whole. Thus any user could either choose between the upstream firms, each of which have distinct advantages (e.g. bundled television, phone, or cable services), or buy internet from any of a large number of competitive, downstream ISPs.

## 7.3 Comparison with functional separation

As discussed in section 2, the Berkman Report praised the notion of functional separation, discussing very successful cases of its application in England and New Zealand. As a policy direction, functional separation—severing physical network infrastructure from the control of (current) incumbent ISPs—might be an effective means of achieving a more competitive Canadian market.

Adapting functional separation into this model would not be difficult and would, in some ways, simplify the model. There would be one or two “upstream” firms, though unlike the upstream firms presented in this paper, these firms would own the infrastructure but sell only to downstream firms, not to consumers. This would necessitate the need for an intermediate market between the upstream firms and multiple downstream firms, with some method of resolving the seller and buyer market power resulting from the very small number of sellers

and buyers.

This could be further enhanced by adapting the observation of Höffler (2007), discussed in section 2, that the competitive benefits of redundant facilities do not outweigh the cost of such redundancy. This would, however, complicate the model as it would require introducing additional cost components to capture the savings of building one network instead of two competing, parallel networks.

## 7.4 Altering marginal cost

Another extension worth exploring, related to the investment extension of 7.1, would be to change the structure of that investment—or perhaps adding a second investment choice—in the model that would allow a firm to make investment that affects its marginal upstream access cost,  $c_{u,i}$ . The regulator-defined price could then either be per-firm (e.g.  $c_{r,i} = \mu c_{u,i}$ ,  $c_{r,j}$  similar) or the mean of upstream access costs ( $c_r = \mu \frac{c_{u,i} + c_{u,j}}{2}$ ). In particular, how this affects the results of section 5.3 would be interesting, since, by basing the regulated price on the marginal cost of access the upstream firms might have an incentive to reduce investment so as to increase the marginal cost (and thus the retail price) of the downstream firm.

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